The Military Multiplier*

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Abstract

How effectively does defense spending translate into military capability? We introduce the *military multiplier*, defined as the percentage increase in military equipment generated by an additional dollar of defense spending. We show that the response of the relative price of defense goods to military buildups is a sufficient statistic for this multiplier: the stronger the price response, the smaller the multiplier. Time-series evidence for the United States shows that defense-sector prices rise sharply in response to military buildups in the post–Cold War period, implying a short-run multiplier of about 0.7, compared with values exceeding 1 during the Cold War. We develop and calibrate a multi-sector network model of the U.S. economy showing that this decline reflects high effective capital reallocation costs associated with the shrinking industrial base.

Keywords: Military buildups, Defense spending, Network model, IO network, investment network, Capital reallocation
JEL-Codes: H56, E62, E23, O41

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1 Introduction

How much military equipment does an additional dollar of defense spending actually buy? We define the military multiplier (*MM*) as the percentage increase in military equipment obtained for an additional dollar of defense spending.[1] This distinction matters because defense commitments and spending targets—such as those within NATO—are typically stated in nominal terms, so even large spending increases need not translate one-for-one into military capabilities. Because defense spending is concentrated in specific sectors of the economy (Cox et al., 2024), a large buildup requires a costly and time-consuming reallocation of resources (Ramey and Shapiro, 1998). When this reallocation bids up the relative price of defense goods, the *MM* falls below one. We estimate that the short-run *MM* is about 0.7 in the post-Cold War period, down from about 1.1 during the Cold War. Put differently, procuring a given amount of military equipment now costs roughly 60 percent more in real terms.

Why has the *MM* fallen so sharply, and why is it below one in the U.S. today? We show that the response of the relative price of defense goods to military buildups is a sufficient statistic for the *MM*: the stronger the price response, the smaller the *MM*. Empirically, in the post–Cold War period the prices of defense goods and manufactured goods relative to the rest of the economy—which account for a large share of defense procurement—rise significantly and persistently following the military spending news shocks compiled by Ramey (2011, 2016). By contrast, the relative price of defense goods fell during the Cold War.

The response of defense prices depends on how easily defense production can expand in response to buildups. We formalize this notion in a multi-sector business cycle model with input-output and investment networks and costly capital reallocation across sectors. These networks matter because a military buildup requires not only final defense production, but also the intermediate inputs and investment goods needed to expand defense capacity. We calibrate the model to both the post-Cold War and Cold War U.S. economies, allowing the input-output network to differ across periods so as to capture changes in the composition of defense production. Quantitatively, however, the main driver of the decline in the *MM* is the shrinking relative size of the industrial base: changes in network structure matter, but their contribution is modest relative to the rise in effective reallocation costs created by deindustrialization.

[1] The *MM* is distinct from the *fiscal multiplier*, which measures the percentage change in aggregate output resulting from an increase in government spending of one unit of output.

To isolate this mechanism, our baseline model assumes that labor is mobile across sectors and production follows a Leontief technology.[2] This keeps the focus on capital as the limiting factor in the expansion of the defense sector. Capital can be expanded either through investment—which takes time and requires inputs from other sectors through the investment network, following Vom Lehn and Winberry (2022)—or through costly reallocation from other sectors, as in Ramey and Shapiro (1998). For a given level of adjustment costs, the size of the defense sector relative to the sectors from which capital can be reallocated is therefore central. We abstract from endogenous markup movements, consistent with our empirical evidence: firms operate under perfect competition and prices rise to signal the scarcity of capacity in the defense sector. A value of *MM* greater than unity arises in the model only if the buildup is accompanied by an exogenous improvement in defense production efficiency, inducing the price of defense goods to *fall*.

We calibrate the model assuming 64 sectors, using input–output data from the Bureau of Economic Analysis and investment network data from Vom Lehn and Winberry (2022). This allows us to provide a granular perspective on the connectivity of the defense sector within both networks and captures shifts in the composition of defense production, including the greater importance of high-technology inputs in modern weapon systems. Importantly, we assume that capital can be reallocated to the defense sector from the 19 manufacturing sectors, while reallocation from other sectors of the economy—such as services—is prohibitively costly. We identify the reallocation costs by targeting the price responses in the manufacturing sectors to military buildups. Using model-based counterfactuals we quantify how much of the decline in *MM* is due to changes in network structure and how much is due to the shrinking manufacturing base. Changes in the production and investment network alone explain only a limited part of the decline in the *MM*, whereas the smaller manufacturing base accounts for most of it. A direct implication is that increases in defense budgets in economies with a small industrial base may have only a limited impact on military capacity.

We also examine the role of policy by computing the *MM* under alternative assumptions about the persistence of the military buildup. This persistence reflects, in a stylized way, whether the buildup is part of a credible long-term strategy. We identify a trade-off: a more persistent buildup reduces the impact

[2]See Atalay (2017) and Boehm et al. (2019) for evidence on very limited short-run substitution of factors of production.

multiplier, as prices initially respond more strongly to the anticipated sustained rise in demand, but it raises the cumulative multiplier over time as investment comes online and expands capacity. By contrast, under a purely transitory one-time increase in military spending, the cumulative multiplier remains essentially flat and stays around 0.8 even after five years.

In light of the Russian invasion of Ukraine and rising geopolitical tensions, many observers point to the economic strength of the European Union—whose economy is roughly ten times the size of Russia's—and argue that expanding its defense capabilities and providing sufficient support for Ukraine should be relatively straightforward (see, for instance, Jensen et al., 2025). Also, recent work by Federle et al. (2025) highlights the critical role that economic resources play in determining the outcomes of wars.

However, this argument overlooks the underlying mechanics of the *MM*. We demonstrate that while the size of an economy and the availability of economic resources are important determinants of military capability, they are not sufficient on their own. Large economies may still lack the capacity to produce military equipment at the required pace. In fact, there is suggestive evidence that recent increases in military spending have, to a significant extent, been absorbed by rising prices. Following the onset of the war in Ukraine in 2022, arms prices accelerated markedly in both Europe and the United States—outpacing the growth of the producer price index, see for example Reuters (2023).

We introduce the notion of the *MM* against the background of a large literature on the fiscal multiplier, which dates back to Keynes (1936), with modern treatments by Barro (1981), Woodford (2011), Auclert et al. (2024), and *many* others. The fiscal multiplier measures the percentage increase in aggregate output in response to an additional unit of government spending. The fundamental concern of this literature is how private expenditure (consumption, investment, and, in an open economy, net exports) responds to an increase in government spending, as this determines its effectiveness in stabilizing the economy—particularly when monetary policy is constrained by the zero lower bound (Christiano et al., 2011). If private expenditure rises in response to additional government spending (is "crowded in"), the fiscal multiplier is greater than one; if it is crowded out, the fiscal multiplier is less than one.

In the context of our analysis, the response of private spending also matters for the *MM*. It is larger when non-government demand for defense-sector goods—whether from the domestic private sector or from foreign governments—is both substantial and price-elastic. In such cases, an increase in government demand tends to crowd out non-government demand, which in turn limits the rise in the price of defense goods and reduces the need to meet additional demand through increased production. It follows directly that, all else being equal, an arms-exporting country will be characterized by a larger *MM*. Quantitatively, however, this channel is modest in the U.S., where arms exports are small relative to domestic military procurement.

We note several caveats. First, as the nature of warfare evolves—such as with the replacement of fighter jets by drones—the sectors involved in the production of defense equipment may change, at least to some extent. This does not invalidate the mechanics of the *MM*, but it may require a recalibration of the model. Second, we put forward a closed-economy model and abstract from the fact that military equipment is sometimes imported. In the limiting case where the domestic economy is small relative to world markets, foreign procurement of military equipment would not affect prices. However, reliance on foreign imports for military equipment introduces its own set of risks. Third, our analysis focuses on the short to medium run and does not account for an endogenous response of defense-sector productivity to defense spending (Ilzetzki, 2024; Antolin-Diaz and Surico, 2025). Instead, we allow for an exogenous improvement in defense production efficiency to account for the observation that the *MM* exceeds unity in the Cold War period. Fourth, our analysis is silent on the optimal level of defense spending or a potential tradeoff between "guns and butter" (Valaitis and Villa, 2025; Marzian and Trebesch, 2026).[3] However, fundamental insights that follow from our analysis remain valid despite these caveats: economic strength alone is not a comprehensive measure of how easily military capabilities can be expanded; the structure of the industry is key, too.

The paper is structured as follows. In the remainder of the introduction, we place the paper in the context of the literature. In the next section, we formally develop the notion of the *MM* and establish some basic relationships. In Section 3, we estimate the impulse response of the defense price index to military buildups. Section 4 introduces the model. We derive results and calibrate the model in Section 5. Based on the calibrated model we run counterfactuals to shed light on the transmission mechanism. A final section concludes.

Related literature. Our work relates to three strands of the literature. First and foremost, it builds on the seminal contribution by Ramey and Shapiro

[3] Alekseev and Lin (2026) study optimal trade taxes in the presence of a military externality due to dual-use goods.

(1998). We extend their approach by incorporating costly capital reallocation into a state-of-the-art multi-sector RBC model with production and investment networks. Our focus also differs: we are concerned with the *MM*, a concept typically not explicitly analyzed in discussions of military buildups, see the recent survey by Ilzetzki (2025). More generally, however, recent work on fiscal policy has increasingly employed multi-sector models. Bouakez et al. (2023) and Acemoglu et al. (2016) show that sectoral linkages amplify and propagate government spending shocks, while Bouakez et al. (2025) emphasizes the importance of the sectoral origin of fiscal shocks. Devereux et al. (2023) and Flynn et al. (2022) extend these insights to regional and international trade. Ramey (2019) surveys the literature on fiscal multipliers, highlighting the distinct effects of military expenditures.

Second, there is the literature on the reallocation of the existing stock of capital (Eisfeldt and Rampini, 2006, 2007; Cooper and Schott, 2013; Rampini, 2019; Wang, 2021; Lanteri and Rampini, 2023). We show that capital reallocation plays a critical role in understanding the effectiveness of military buildups. We also contribute to the literature examining the macroeconomic effects of fiscal policy under factor immobility and sectoral heterogeneity. Cardi et al. (2020) and Proebsting (2022) show that costly labor mobility helps explain the macroeconomic responses to fiscal shocks. In contrast, we focus on the costly mobility of capital and explore its implications for the effectiveness of military buildups. Relatedly, Fordham (2003) documents that the relative price of defense goods responds to labor-market conditions and industrial concentration, consistent with the supply-side mechanism we formalize. More broadly, we relate to the literature on the aggregate effects of reallocation shocks under frictions. Phelan and Trejos (2000) and Ferrante et al. (2023) show that reallocation can have significant aggregate effects on output and inflation, highlighting the macroeconomic costs of shifting resources across sectors. Relative to the existing military-buildup literature, our contribution is to show how these sectoral frictions determine not only price responses, but also how much military equipment is ultimately obtained from a given increase in spending.

Finally, we relate to the literature on sectoral shock propagation in multi-sector real business cycle (RBC) economies (Horvath, 2000; Foerster et al., 2011; Atalay, 2017), as well as to studies emphasizing input-output linkages in production (Long and Plosser, 1983; Acemoglu et al., 2012; Baqaee and Farhi, 2019). We contribute to this literature by introducing a network of costly capital reallocation and demonstrating that it plays a crucial role in determining the effectiveness of sectoral government spending.

2 Military multiplier basics

Spending a certain amount—say, a fraction of GDP—on defense goods is not an end in itself. Rather, the objective of defense spending is to enhance military capacity by procuring a certain quantity of defense goods. Against this background, we introduce a concept of *military multiplier* (*MM*) aiming to capture the ease with which the economy can convert the resources spent on defense into the military equipment produced. Then we zoom in on the market for military goods, maintaining a partial equilibrium perspective which we relax later.

2.1 Definition

To fix ideas, we consider an economy where the government only purchases military goods, G_t, from a specific sector while private consumption, C_t, and investment, I_t, have the same composition as GDP, Y_t, such that, with price indices appropriately defined, we have: $P_t^Y Y_t = P_t^G G_t + P_t^Y C_t + P_t^Y I_t$. We deflate all expenditure components with P_t^Y and define P_t^G as the relative price of government spending in units of output, which we use as a numéraire good. Further, using hats to express the change of variables in terms of steady-state output, e.g., $\hat{g}_t = \frac{G_t - G}{Y}$; we write the percentage change of output as follows:

$$\hat{y}_t = \hat{x}_t + \hat{c}_t + \hat{i}_t, \tag{2.1}$$

where $\hat{x}_t \equiv \hat{p}_t + \hat{g}_t$ is the percentage change in military spending measured in units of output and, assuming that relative prices are unity in steady state, $\hat{p}_t \equiv \frac{G}{Y} p_t$; here and in what follows we use letters without hats to measure the percentage deviation of a variable from its steady-state value. Given this, we define two different multipliers.

- The **fiscal multiplier** is the percentage change of real output per percentage increase in government spending, measured in units of output:

$$M \equiv \frac{\hat{y}_t}{\hat{x}_t} = \frac{\hat{x}_t + \hat{c}_t + \hat{i}_t}{\hat{x}_t}.$$

- The **military multiplier** is the percentage change of real military equipment per percentage increase in government spending, measured in units of output:

$$MM \equiv \frac{\hat{g}_t}{\hat{x}_t} = 1 - \frac{\hat{p}_t}{\hat{x}_t}. \tag{2.2}$$

Several remarks are in order. First, as we compute the *MM*, we disregard the response of consumption and investment, which are key for understanding the fiscal multiplier (Barro, 1981; Baxter and King, 1993). Second, the *MM* will differ from unity only as long as $\hat{p}_t \neq 0$. Expression (2.2) also shows that the response of $\hat{p}_t$ is a sufficient statistic for backing out the military multiplier. Third, in one-sector models we have $\hat{p}_t = 0$, such that $MM = 1$.

Finally, we note that we share the modern perspective on the multiplier. The traditional Keynesian fiscal multiplier rests on the notion that, with fixed prices, private demand is entirely determined by current income and output is fully demand-driven. In contrast, modern dynamic macroeconomic models typically assume incomplete price stickiness. As a result, private demand depends not only on current income but also on intertemporal prices—namely, interest rates—which adjust in response to government spending shocks. Thus, price adjustments are essential for determining the multiplier: relative prices in the case of the military multiplier, and intertemporal prices in the case of the fiscal multiplier.

Below, we also report the *cumulative MM*, adapting the definition of Mountford and Uhlig (2009) or, equivalently the "present value multiplier," see also Ramey (2019). Formally, we compute the cumulative value of the response of real military spending over time divided by the cumulative value of the government spending response over time, measured in units of output, to the shock:[4]

$$\text{Cumulative } MM \text{ at lag k } = \frac{\sum_{j=0}^{k} g_j}{\sum_{j=0}^{k}(g_j + p_j)} = 1 - \frac{\sum_{j=0}^{k} p_j}{\sum_{j=0}^{k} x_j}. \tag{2.3}$$

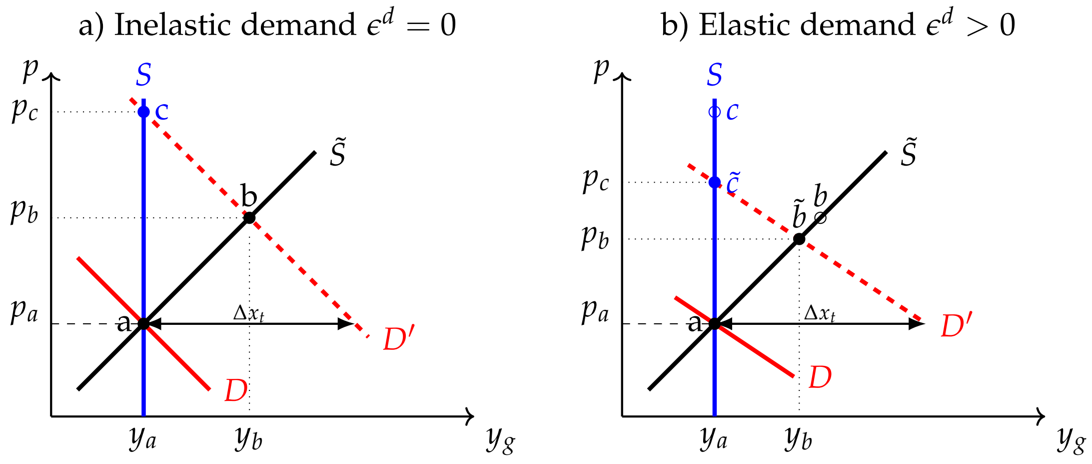
2.2 The market for defense goods in partial equilibrium

The *MM*, as defined above, depends on how the price of defense goods responds to the increase in spending triggered by the military buildup. From a partial equilibrium perspective on the market for defense goods, the price response is determined solely by the price elasticities of both supply and demand. In what follows, we formalize this basic insight to set the stage for the subsequent sections.

In line with our full model presented below, we allow for the possibility that the government is not the sole buyer of defense goods. There may also be private-sector demand—originating either domestically or abroad—with price elasticity denoted by ϵ^d. Using ϵ^s to denote the price elasticity of supply, we

[4]For simplicity, we do not discount spending that takes place later in time.

Figure 1: Equilibrium in the market for defense goods



Notes: Left (right) panel assumes inelastic (elastic) private-sector demand. Downward-sloping D-lines represent the transformed demand curve, see Equation (2.7). Supply curves S, see Equation (2.5), are shown for two cases: perfectly inelastic (vertical) and elastic (upward sloping).

write demand and supply in the market for defense goods as follows:

$$y_{g,t} = -\epsilon^d \cdot p_t + g_t, \tag{2.4}$$

$$y_{g,t} = \epsilon^s \cdot p_t, \tag{2.5}$$

where all variables are expressed in percentage changes from steady state. Substituting in (2.2), we obtain for the MM:

$$MM = \left[1 + \frac{1}{\epsilon^d + \epsilon^s}\right]^{-1}, \tag{2.6}$$

that is, the MM is increasing in both, the elasticity of demand and supply: The more elastic both sides of the market, the less strong the increase in prices, and the more effective the military buildup.

It is instructive to rewrite the demand function (2.4) in such a way that it depends on the government's purchases in terms of the numéraire good:

$$y_{g,t} = -(1 + \epsilon^d) \cdot p_t + x_t. \tag{2.7}$$

In this way, we capture the observation that military buildups typically specify a policy target in terms of the numéraire good x_t, for example, by specifying a certain spending target in percent of GDP. The extent to which military purchases change in real terms then depends on the price response. Figure 1 illustrates this graphically. Both panels of the figure show the equilibrium in the market for defense goods, with prices measured along the vertical axis and quantities along the horizontal axis. The downward-sloping D-line represents equation (2.7) and shifts to the right as a result of the military buildup, specified in terms of the numéraire good, indicated by the dashed D'-line.

In the left panel we assume that private demand is perfectly inelastic and distinguish two scenarios for the price-elasticity of supply, indicated by the vertical blue line (inelastic supply) and the upward sloping black line (elastic supply). The implications are straightforward. If supply is inelastic, the outward shift in demand is fully absorbed by prices. There is no additional production of military goods (point c). The *MM* is zero. If instead, supply is elastic, both production and prices increase (point b).

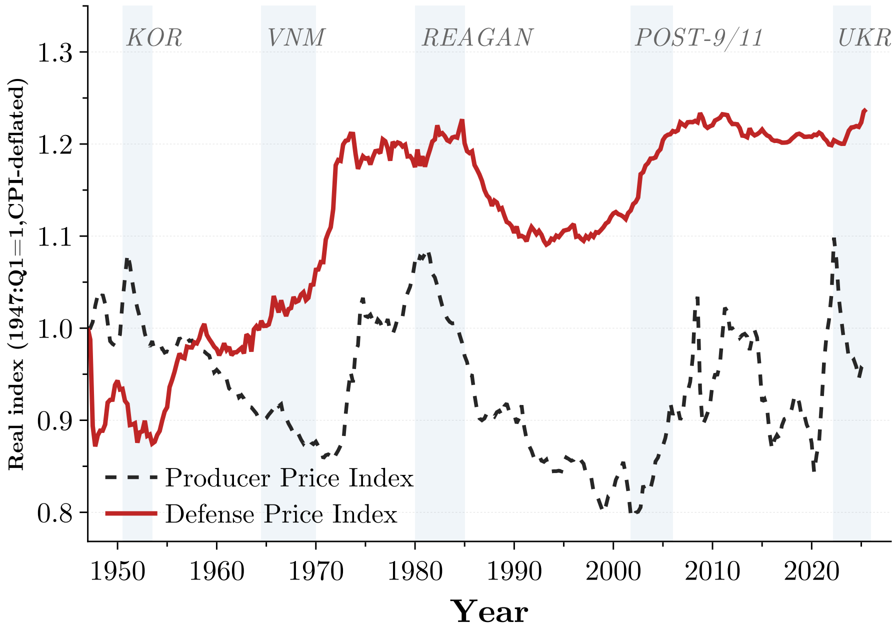
In the right panel, we consider the case of a non-zero demand elasticity, meaning that as prices rise, private purchases of defense goods are crowded out. As a result, the D-line is flatter, and as it shifts outward, prices increase less than in the case of inelastic demand—regardless of the supply scenario. Note that in this case, production (measured along the horizontal axis) also increases less, reflecting the decline in private-sector demand, which now makes room for the government's additional purchases. As emphasized above, what matters for the *MM* is the price response. This response is weaker the more elastic both demand and supply are.

Against this background, it is essential to understand the underlying determinants of both supply and demand in the market for defense goods. To achieve this, we must shift from a partial to a general equilibrium perspective, since the adjustment of both supply and demand ultimately depend on reallocation across sectors in response to the buildup. The multi-sector business cycle model in Section 4 provides such a perspective. In the next section, however, we first present evidence on how prices respond to military buildups. We show that defense-sector prices rise strongly in the post–Cold War period but fall during the Cold War, implying a substantial decline in the military multiplier over time.

3 Time-series evidence

The previous section established that the effectiveness of defense spending depends inversely on the response of defense-sector prices to a military buildup: larger increases in relative prices imply a lower multiplier. Against this back-

Figure 2: Military expenditure prices vs. producer prices



Notes: Implicit price deflator for federal national defense expenditures and gross investment (BEA NIPA Table 1.1.9) and producer price index (PPI), both normalized by the GDP deflator.

ground, we use U.S. time-series data to assess whether the relevant prices respond significantly to military spending shocks.

Before turning to the formal analysis, Figure 2 provides a first look at the data. It plots the implicit price deflator for federal national defense consumption expenditures and gross investment alongside the producer price index (PPI) from 1947 onward. The defense deflator, published by the Bureau of Economic Analysis as part of the National Income and Product Accounts (NIPA Table 1.1.9), captures price changes for the entire basket of goods and services procured by the federal government for national defense—including weapons systems, military equipment, construction, and operations.[5] Two features stand out. First, military prices rise more steeply than the PPI over the full sample—by the end of the sample, the defense deflator has increased by almost 30 percent more—indicating that the relative price of defense goods has trended upward persistently. Second, major military buildups—the wars in Korea, Vietnam, and the post-9/11 period, as well as the recent Ukraine-related buildup—are visible, consistent with the notion that expanding defense procurement drives up prices in the sectors involved. In what follows, we use a formal identification

[5]The deflator measures constant-quality price change using asset-specific indexes (including BLS PPIs, construction cost indexes, and hedonic indexes for some high-technology equipment).

strategy to establish a causal link between military spending shocks and price movements.

Specifically, we estimate the responses of several variables to defense news as used by Ramey and Zubairy (2018).[6] For this purpose we run local projections in the spirit of Ramey and Shapiro (1998), for each horizon h:

$$y_{t+h} = \alpha_0 + \alpha_1 t + \sum_{i=1}^{8} b_i y_{t-i} + \sum_{i=0}^{8} c_i D_{t-i} + \varepsilon_t, \tag{3.1}$$

where y_{t+h} denotes the outcome variable of interest h periods ahead, and D_{t-i} is the military spending news shock. By estimating a distinct regression for each horizon, this approach allows us to trace out the impulse response without relying on a fully specified dynamic model. We provide details on the data in Appendix A.1.

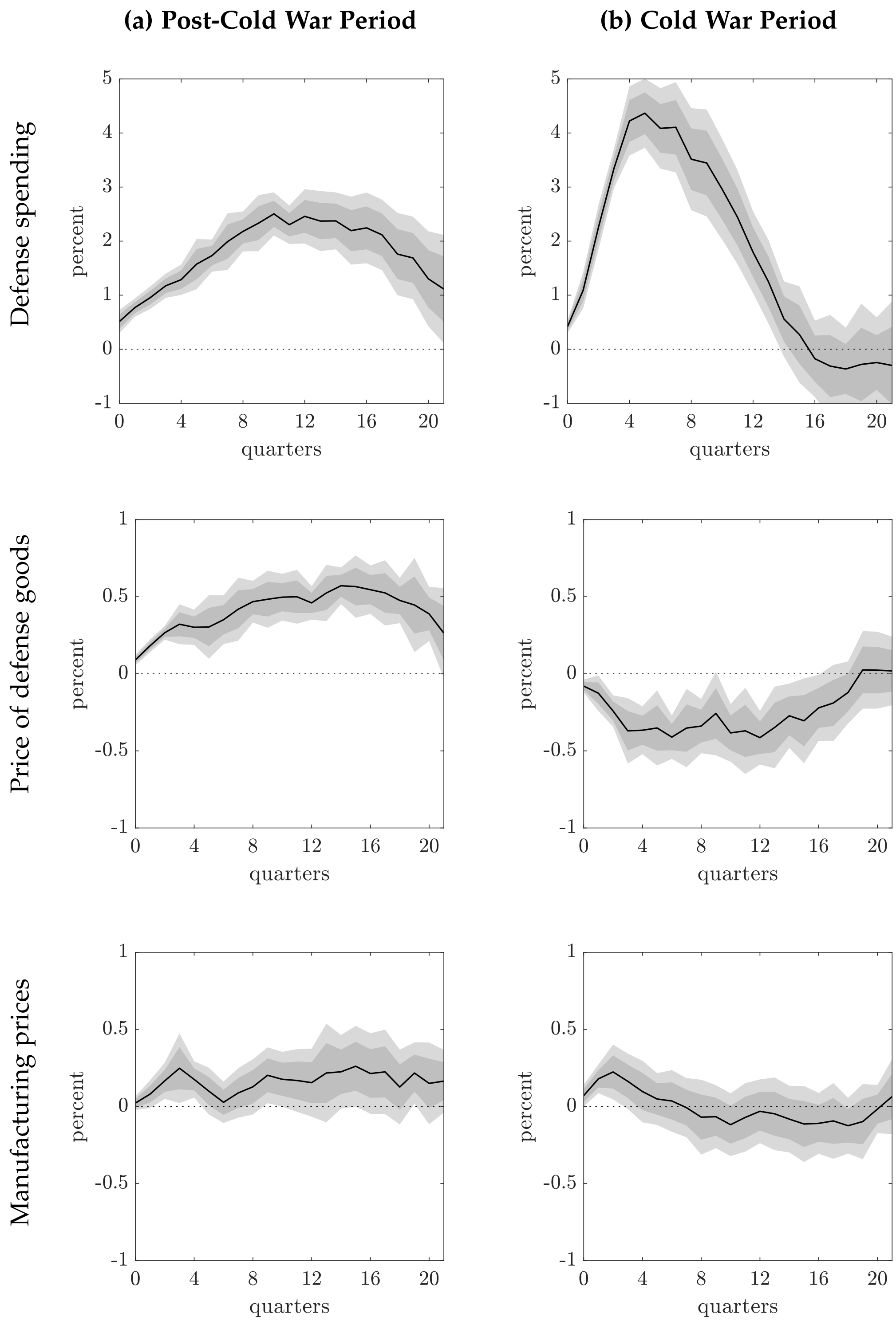
Figure 3 shows the results. The panels in the left column present our baseline estimates for the post–Cold War period (1991–2018), which we contrast with the corresponding estimates for the Cold War period shown in the right column. In each panel, the horizontal axis measures time in quarters, and the vertical axis measures percentage deviations from the pre-shock level. Solid lines represent point estimates, while shaded areas indicate 68 and 90 percent confidence intervals.

Consider first the response of defense spending, shown in the top panels. Defense spending increases strongly over time in a hump-shaped manner, indicating that the buildup unfolds gradually. In the post–Cold War period, the average buildup increases defense spending by 2 percent and is quite persistent. By contrast, during the Cold War period, defense spending rises more sharply by up to 4 percent but reverts more quickly. Our main focus is the response of defense-sector prices, shown in the middle panels. These prices increase strongly and persistently—by almost 0.5 percent—in the post–Cold War period, whereas defense-goods prices fall during the Cold War period.

The bottom panels show the response of manufacturing-goods prices. The manufacturing sector is the largest recipient of Department of Defense purchases, accounting for about 40 percent (see Table A.15 in the Online Appendix of Cox et al. (2024)). Again we find a persistent increase during the post-Cold War period of around 0.3 percent. In the Cold War period, by contrast, the real price of manufacturing goods increases for about four quarters, peaking

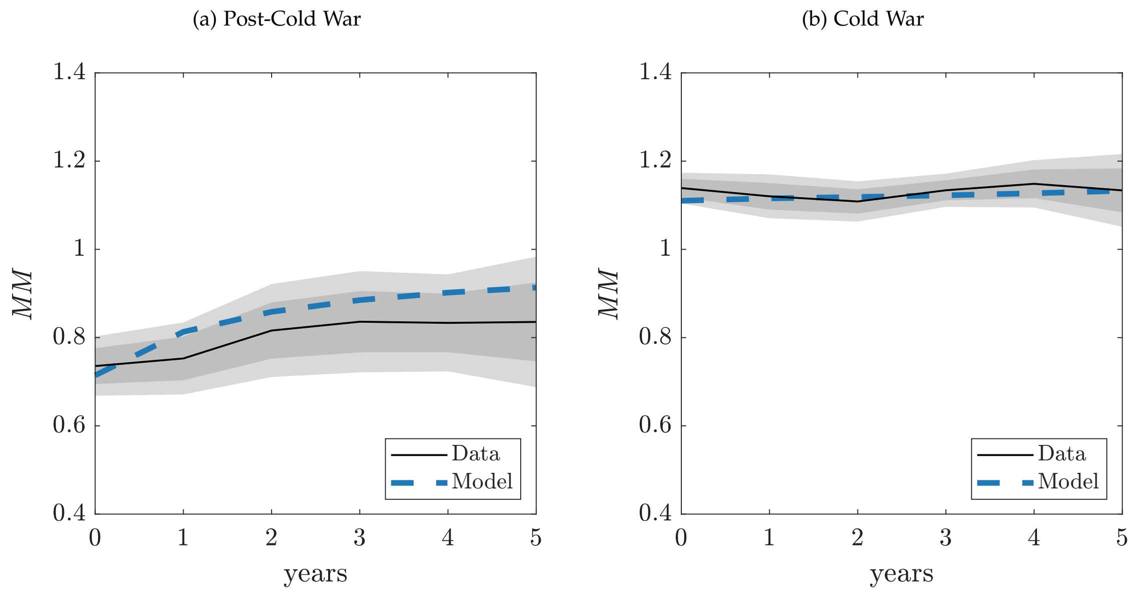
[6] Ramey and Zubairy (2018) use the shock originally described in Ramey (2016), which runs up to 2015. We use outcome variables up to 2018 for the horizons $t+h$.

Figure 3: Response to military buildups



Notes: Quarterly response of government defense spending, prices of defense goods (deflated with GDP deflator) and manufacturing prices (deflated with GDP deflator) to military spending news from Ramey (2016) during Post-Cold War period (1991–2018) and Cold War period (1947–1990). Black solid lines represent the point estimates while shaded areas indicate 68 and 90 percent confidence bounds.

Figure 4: Cumulative military multiplier



Notes: Cumulative military multiplier over the first 5 years after the military buildup shock. Left panel: *MM* in Post-Cold War Period (data) and service economy model (small manufacturing sector). Right panel: *MM* in Cold War Period (data) and in industrial economy model (large manufacturing sector). Black solid lines represent the point estimates while shaded areas indicate 68 and 90 percent confidence bounds.

at approximately 0.2 percent, before reverting quickly. This result is consistent with Ramey and Shapiro (1998), who also report positive price responses of manufacturing to military news using data through 1996.

Using the sufficient statistic established in the previous section, we can directly compute the cumulative military multiplier and assess how it has changed over time. Formally, we compute $MM(k) = 1 - \frac{\sum_i^k p_{t+i}}{\sum_i^k x_{t+i}}$ and show the results for both samples in Figure 4.[7] We find a military multiplier in the first year of the buildup of 0.7 in the post-Cold War period, but larger than 1 in the Cold War period—consistent with the drop in prices documented for this period above. The cumulative multiplier in the post-Cold War period slowly increases over time but remains smaller than unity throughout, peaking at 0.85.

Before rationalizing these findings with our model, we establish additional

[7]We follow Ramey and Zubairy (2018) to obtain standard errors using a local projection instrumental variables (LP-IV) approach and estimate a regression of the form:
$\sum_{j=0}^{h} p_{t+j} = \alpha_0 + \alpha_1 t + b_i \sum_{j=0}^{h} x_{t+j} + \sum_{i=1}^{4} c_i X_{t-i} + \varepsilon_t$,
where x_{t-i} are instrumented with the news spending shocks, and X_{t-i} is a vector of controls, including four lags of p_t, x_t, as well as of the shock series.

evidence based on more granular data. First, the evidence presented above relies on the defense deflator, which captures military expenditures in a broad sense. For the post–Cold War period, however, a more narrowly defined price index for arms and ammunition is available. We find that this index is more than four times as responsive as the defense deflator (see Appendix A.2). Hence, the *MM* associated with military buildups focused on rearmament are likely to be smaller than those implied by our baseline estimates.

Second, we estimate the impulse response of markups to government spending shocks following the approach of Nekarda and Ramey (2020). We find no significant increase in markups in the post-Cold War sample and only a mild increase during the Cold War period. We report these results in Appendix A.2. Based on this evidence, we reject the notion that the price response shown in Figure 3, and thus our estimates of *MM*, are driven by changes in markups.

4 A multi-sector economy

Our setup is based on the multi-sector real business cycle model with input-output and investment networks of Vom Lehn and Winberry (2022). To study the effects of military buildups, we extend their model in two dimensions. First, we allow for costly reallocation of capital across sectors. Second, we introduce sectoral government spending following Cox et al. (2024). These extensions allow us to model the short-run sectoral dynamics due to military buildups. Note that the model assumes perfect competition as a result of which markups are unresponsive to government spending, consistent with evidence by Nekarda and Ramey (2011).

4.1 Households

A representative household maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \frac{L_t^{1+\gamma}}{1+\gamma} \right]$$

subject to the budget constraint $C_t + Q_{t,t+1}B_{t+1} = B_t + W_t L_t + T_t$. Here β and γ are positive constants and E_0 is the expectation operator. C_t is consumption, L_t is hours worked, B_t is bond holdings, $Q_{t,t+1}$ is the stochastic discount factor, W_t is the wage rate, T_t are lump-sum payments, including firm profits and taxes. We normalize the price of consumption to one; all other prices (and wages) are expressed in relative terms. Ruling out Ponzi schemes, the first-order conditions are given by:

$$Q_{t,t+1} = \beta \cdot E_t \frac{C_t}{C_{t+1}}, \quad \text{(Euler equation)} \tag{4.1}$$

$$L_t^{\gamma} = \frac{W_t}{C_t}. \quad \text{(Labor supply)} \tag{4.2}$$

Sectoral consumption demand. Consumption C_t is an aggregate of sector-specific consumption goods: $C_t = \bar{b} \prod_{i=1}^{N} C_{t,i}^{b_i}$ where N is the number of sectors and $C_{t,i}$ is consumption of sector-i good. $\sum_{i=1}^{N} b_i = 1$ and $\bar{b} = [\prod_{i=1}^{N} b_i^{b_i}]^{-1}$ is a normalizing constant. Let $P_{t,i}$ be the price of sector-i good. Then, sector-specific consumption demand and the consumer price index are given by:

$$P_{t,i} C_{t,i} = b_i \cdot C_t, \quad \text{(Sector } i \text{ consumption demand)} \tag{4.3}$$

$$\prod_{i=1}^{N} P_{t,i}^{b_i} = 1. \quad \text{(Consumer price index)} \tag{4.4}$$

Sectoral labor supply. Total hours worked consists of labor supplied to each of N sectors, that is

$$L_t = \sum_{i=1}^{N} L_{t,i}, \quad \text{(Labor aggregation)} \tag{4.5}$$

where $L_{t,i}$ is labor supplied to sector i. Note that in the baseline model, labor is perfectly mobile across sectors. Additionally, we consider an alternative specification with sector-specific labor.

4.2 Production

Each sector consists of a set of identical perfectly competitive firms. Firms in sector i produce output $Y_{t,i}$ based on a sector-specific CRS production technology:

$$Y_{t,i} = F_i(A_{t,i}, \hat{K}_{t,i}, L_{t,i}, ..X_{t,ij}, ...)$$

where $\hat{K}_{t,i}$ is capital input, $L_{t,i}$ labor input, $X_{t,ij}$ sector-j output used as intermediate input in sector i; $A_{t,i}$ is sector-specific productivity. Our analysis is based on general CES-type specification:

$$Y_{t,i} = \bar{\omega} A_{t,i} \cdot \left[\theta_i \left(\alpha_i \hat{K}_{t,i}^{1-1/\epsilon_i} + (1-\alpha_i) L_{t,i}^{1-1/\epsilon_i} \right) + (1-\theta_i) \sum_{j=1}^{N} \omega_{ij} X_{t,ij}^{1-1/\epsilon_i} \right]^{\frac{\epsilon_i}{\epsilon_i - 1}}$$

It nests the Leontief case for $\epsilon_i \to 0$):

$$Y_{t,i} = \bar{\omega} A_{t,i} \cdot \min\left\{ \frac{1}{\theta_i} \min\left(\frac{\hat{K}_{t,i}}{\alpha_i}, \frac{L_{t,i}}{1-\alpha_i} \right), \frac{1}{1-\theta_i} \min_j \left(\frac{X_{t,ij}}{\omega_{ij}} \right) \right\}$$

and the Cobb-Douglas case for $\epsilon_i \to 1$:

$$Y_{t,i} = \bar{\omega} A_{t,i} \cdot \left(\hat{K}_{t,i}^{\alpha_i} L_{t,i}^{1-\alpha_i} \right)^{\theta_i} \cdot \left(\prod_{j=1}^{N} X_{t,ij}^{\omega_{ij}} \right)^{1-\theta_i}.$$

Given sector-specific capital costs, $r_{t,i}$, the optimal choice of inputs satisfies the first order conditions:

$$r_{t,i}^{\epsilon_i} \hat{K}_{t,i} = \alpha_i \theta_i \cdot P_{t,i}^{\epsilon_i} Y_{t,i}, \qquad \text{(Sector } i \text{ capital demand)} \quad (4.6)$$

$$W_{t,i}^{\epsilon_i} L_{t,i} = (1-\alpha_i)\theta_i \cdot P_{t,i}^{\epsilon_i} Y_{t,i}, \qquad \text{(Sector } i \text{ labor demand)} \quad (4.7)$$

$$P_{t,j}^{\epsilon_i} X_{t,ij} = (1-\theta_i)\omega_{ij} \cdot P_{t,i}^{\epsilon_i} Y_{t,i}. \qquad \text{(Sector } i \text{ intermediate input demand)} \quad (4.8)$$

Given perfect competition, the sector-specific price is equal to marginal costs and given by:

$$P_{t,i} = \frac{1}{A_{t,i}} \left[\theta_i \left(r_{t,i}^{\alpha_i} W_t^{1-\alpha_i} \right)^{1-\sigma_i} + (1-\theta_i) \left(\prod_{j=1}^{N} P_{t,j}^{\omega_{ij}} \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}}. \quad (4.9)$$

In the quantitative analysis, we also consider the Leontief production technology, eliminating the substitutability across factors.

4.3 Investment goods

In each sector, investment goods are produced under perfect competition based on a sector-specific CRS technology, which combines sector-specific goods. Investment in sector i is given by

$$I_{t,i} = \bar{\lambda} \prod_{j=1}^{N} I_{t,ij}^{\lambda_{ij}},$$

where $I_{t,ij}$ is sector-j output used to produce investment in sector i, $\bar{\lambda}$ is a normalizing constant. Given the sector-specific investment price $P_{t,i}^{I}$, a producer of investment goods chooses inputs to maximize profits, yielding the following sector-specific investment demand:

$$P_{t,j} I_{t,ij} = \lambda_{ij} P_{t,i}^{I} I_{t,i}. \quad \text{(Sector-}i\text{ investment inputs demand)} \quad (4.10)$$

The price of sector-i investment good is then given by

$$P_{t,i}^{I} = \prod_{j=1}^{N} P_{t,j}^{\lambda_{ij}}. \quad \text{(Sector } i \text{ investment good price)} \quad (4.11)$$

4.4 Sectoral capital accumulation and reallocation

Each sector accumulates sector-specific capital, which it rents to firms in its own sector i. Capital is obtained either through intersectoral reallocation or through new investment using a sector-specific investment good, so as to maximize the expected present value of profits:

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} \left[r_{t,i}\hat{K}_{t,i} - P^I_{t,i} I_{t,i} - \sum_{j=1}^{N} P^o_{t,ij} R_{t,ij} \right],$$

where $R_{t,ij}$ capital reallocated from sector j to sector i and $P^o_{t,ij}$ price of this capital.

While the new investment becomes available with a lag, the reallocated capital becomes available immediately. Let $K_{t-1,i}$ be sector-i capital at the beginning of period t and total capital reallocated towards sector i be given by $R_{t,i} = \sum_{j=1}^{N} R_{t,ij}$, then the capital available for production at time t is

$$\hat{K}_{t,i} = K_{t-1,i} + R_{t,i} - \underbrace{\frac{1}{2}\sum_{j=1}^{N} \phi_{ij} R^2_{t,ij}}_{\text{realloc. cost}}. \quad \text{(Sector } i \text{ available capital)} \tag{4.12}$$

In the expression above, the third term on the RHS captures the reallocation costs paid by sector-i firms for reallocating capital from each sector. Capital accumulation is given by

$$K_{t,i} = (1-\delta)\hat{K}_{t,i} + I_{t,i} \quad \text{(Sector } i \text{ capital accumulation)} \tag{4.13}$$

The first-order conditions of the firms' optimization problem are given by:

$$P^I_{t,i} = E_t Q_{t,t+1} \left[r_{t+1,i} + (1-\delta) P^I_{t+1,i} \right], \qquad \text{(Investment price dyn.)} \tag{4.14}$$

$$P^o_{t,ij} = \left[r_{t,i} + (1-\delta) P^I_{t,i} \right] \cdot (1 - \phi_{ij} R_{ij}). \qquad \text{(Reallocation price)} \tag{4.15}$$

Reallocation between each pair of sectors implies the following reallocation constraints

$$R_{t,ij} = -R_{t,ji} \qquad \text{(Reallocation quantity symmetry)} \tag{4.16}$$

$$P^o_{t,ij} = P^o_{t,ji} \qquad \text{(Reallocation price symmetry)} \tag{4.17}$$

Reallocation demand. The sector-specific price of old capital in sector i is given by

$$P^o_{t,i} = r_{t,i} + (1-\delta) P^I_{t,i}. \quad \text{(Sector-}i\text{ old capital price)} \tag{4.18}$$

Then equation (4.15) becomes $P^o_{t,ij} = P^o_{t,i}(1 - \phi_{ij} R_{ij})$. Using the reallocation constraints (4.16) and (4.17), we get the sector-pair specific reallocation as

$$R_{t,ij} = \frac{P^o_{t,i} - P^o_{t,j}}{\phi_{ij} P^o_{t,i} + \phi_{ji} P^o_{t,j}}. \tag{4.19}$$

That is, capital is reallocated from j to i as long as $P^o_{t,i} > P^o_{t,j}$ and the reallocation amount is decreasing in reallocation cost parameters ϕ_{ij} and ϕ_{ji}. Total capital reallocated towards sector i is given by:

$$R_{t,i} = \sum_{j=1}^{N} R_{t,ij} = \sum_{j=1}^{N} \frac{P^o_{t,i} - P^o_{t,j}}{\phi_{ij} P^o_{t,i} + \phi_{ji} P^o_{t,j}}. \quad \text{(Capital reallocation)} \tag{4.20}$$

4.5 Government spending and resource constraints

The government purchases goods from each sector, $G_{t,i}$, which are exogenously determined and financed via lump-sum taxation. The government budget constraint is balanced in every period. The resource constraint on output in sector i implies that

$$Y_{t,i} = C_{t,i} + \sum_{j=1}^{N} X_{t,ji} + \sum_{j=1}^{N} I_{t,ji} + G_{t,i}. \quad \text{(Sector } i \text{ resource constraint)} \tag{4.21}$$

That is, sector-i output is either consumed by household, used as intermediate input in production of output and investment goods, or consumed by the government.

4.6 Steady state

We solve the model using a log-linear approximation around the steady state, in which sectoral prices are unity and no reallocation takes place; we use variables without time subscript refer to steady state values and provide further details in Appendix B. In what follows, we define several steady-state ratios to which we refer below.

We use W to denote the input-output matrix, such that $W(i,j) = \omega_{ij}$. The share of intermediate inputs sourced from sector j in sector j production is given by $\frac{X_{ij}}{Y_i} = (1-\theta_i)\omega_{ij}$. The steady state interest rate is $r_i = \frac{1}{\beta} - (1-\delta) = r$. Capital shares are $\frac{K_i}{Y_i} = \frac{1}{r}\alpha_i\theta_i$ and investment shares are $\frac{I_i}{Y_i} = \frac{\delta}{r}\alpha_i\theta_i$; where θ_i is the share of primary inputs in production and α_i is the capital share among these. We use W_λ to denote the investment-production matrix such that $W_\lambda(i,j) = \lambda_{ij}$. The share of inputs in the production of sector-i investment goods sourced from sector j is given by $\frac{I_{ij}}{I_i} = \lambda_{ij}$. The consumption shares are $\frac{C_i}{C} = b_i$. We define the steady state government spending shares as $g_i = \frac{G_i}{C}$.

We may then define the ratio of sectoral sales to aggregate consumption as $\xi_i = \frac{Y_i}{C}$. Then, from the sectoral resource constraint we obtain

$$\boldsymbol{\xi} = [I - W'(I - I_\theta) - \frac{\delta}{r} W'_\lambda I_\alpha I_\theta]^{-1} \cdot (\boldsymbol{b} + \boldsymbol{g}), \tag{4.22}$$

where I_x denotes a diagonal matrix with vector $\boldsymbol{x}$ on the main diagonal, I is the identity matrix, and bold letters denote column vectors of sector-specific variables $\boldsymbol{x_t} = [x_{t,1}, ... \ x_{t,n}]'$.

For our analysis below, we also define a set of private uses of defense-sector goods: $F = \{C_D, X_{iD}.., ..I_{iD}..\}$, that is, all uses excluding government purchases. The private uses span three categories: private consumption, intermediate inputs, and investment inputs. We define private use shares of defense-sector good as $v_f^D = \frac{f}{Y_D}$ where $f \in F$. In particular, $v_C^D = \frac{C_D}{Y_D} = \frac{b_D}{\xi_D}$, $v_{X_{iD}}^D = \frac{X_{iD}}{Y_D} = \frac{(1-\theta_i)\omega_{iD}}{\xi_D}$ and $v_{I_{iD}}^D = \frac{I_{iD}}{Y_D} = \lambda_{iD} \frac{\delta}{r} \frac{\alpha_i \theta_i}{\xi_i \xi_D}$.

5 What determines the military multiplier?

We use the model to study the determinants of the military multiplier. First, we characterize the demand and supply elasticities in the defense-goods market implied by the model's structural parameters, notably reallocation costs and sector size. Second, we calibrate the model to key features of the post–Cold War economy and the time-series evidence in Section 3. Third, we present results for the calibrated model. We show, in particular, that the *MM* is well below unity because the manufacturing sector's moderate size implies that effective reallocation costs are high.

5.1 The market for military goods in general equilibrium

The *MM* equals unity if either the price elasticity of demand or supply is infinite, and declines as these elasticities fall, see again expression (2.6) above. In the limiting case of a demand elasticity of zero, the *MM* goes to zero, whenever the supply elasticity approaches zero. The partial equilibrium perspective in Section 2.2 treats these elasticities as primitives. In the general equilibrium model introduced above, they instead emerge as functions of the economy's use shares of defense-sector goods, the degree of factor substitutability in the defense sector, and the costs of reallocating capital across sectors.

To show this formally, we proceed as follows. We (i) abstract from the input–output network by setting $\theta_i = 1$ and $\alpha_i \to 1$ for all i; (ii) assume that the defense sector uses its own output as the investment good, $\lambda_{DD} = 1$, while investment goods in all other sectors are identical to the consumption good, that is, $\lambda_{ij} = b_j$ for all i; (iii) abstract from discounting ($\beta = 1$) and assume full capital depreciation ($\delta = 1$); (iv) rule out capital reallocation between non-defense sectors ($\phi_{ij} \to \infty$ for $i, j \neq D$); and (v) assume that reallocation costs accrue only in non-defense sectors, so that $\phi_{Dj} = 0$ while $\phi_{jD} > 0$.

Under assumptions (i)–(v), which we relax in the quantitative analysis below, we derive expressions for the price elasticities of demand and supply for defense-sector goods as functions of the model's structural parameters. We summarize the result in the following proposition.

Proposition 1. *Consider the multi-sector economy introduced in Section 4. Under assumptions (i)-(v), the price elasticity of demand for defense-sector goods is given by sum of private use shares in the defense sector's total output:*

$$\epsilon^d = \sum_{f \in F} v_f^D. \tag{5.1}$$

The price elasticity of supply for defense goods, in turn, is approximately given by:

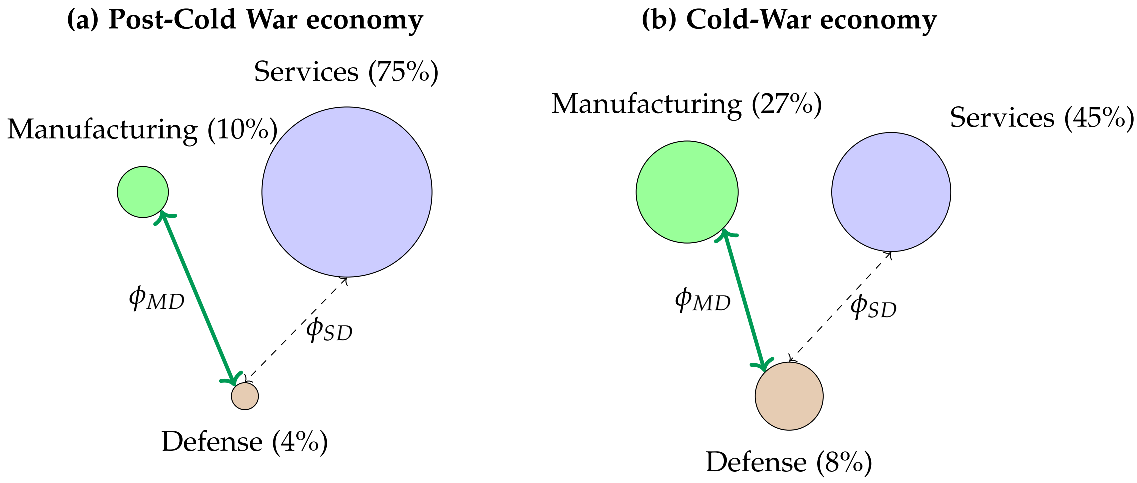
$$\epsilon^s \approx \underbrace{\epsilon_D \cdot \frac{1 - \alpha_D}{\alpha_D}}_{\text{factor substitution}} + \underbrace{\sum_{j=1}^{N} \frac{1}{1 + \tilde{\phi}_{Dj}} \cdot \frac{\xi_j}{\xi_D}}_{\text{capital reallocation}}; \tag{5.2}$$

where $\tilde{\phi}_{jD} = \phi_{jD} R_{Dj,t}$ are the marginal costs of reallocating capital from sector j to the defense sector.

We provide a formal proof in Appendix B.3. The result is intuitive: The demand elasticity, ϵ^d, increases in the private-use share of defense-sector goods, because it governs the scope for crowding out. The supply elasticity, ϵ^s, in turn, increases with factor substitutability, ϵ_D, the labor share in the production of defense goods relative to capital, and decreases with the sum of reallocation costs weighted by the size of the donor sector relative to size of the defense sector, $\tilde{\phi}_{Dj}$.

To explore further how size-weighted reallocation costs determine the supply elasticity, we focus on the Leontief case ($\epsilon_D \to 0$) and consider a stylized

Figure 5: Industry and military size in the US



Notes: Manufacturing and Military shares are from NIPA tables. Cold War economy is year 1950. Post-cold War economy is year 2020.

three-sector economy which features, in addition to the defense sector, a sector *Manufacturing* and a sector *Services*. Then, expression (5.2) simplifies as follows:

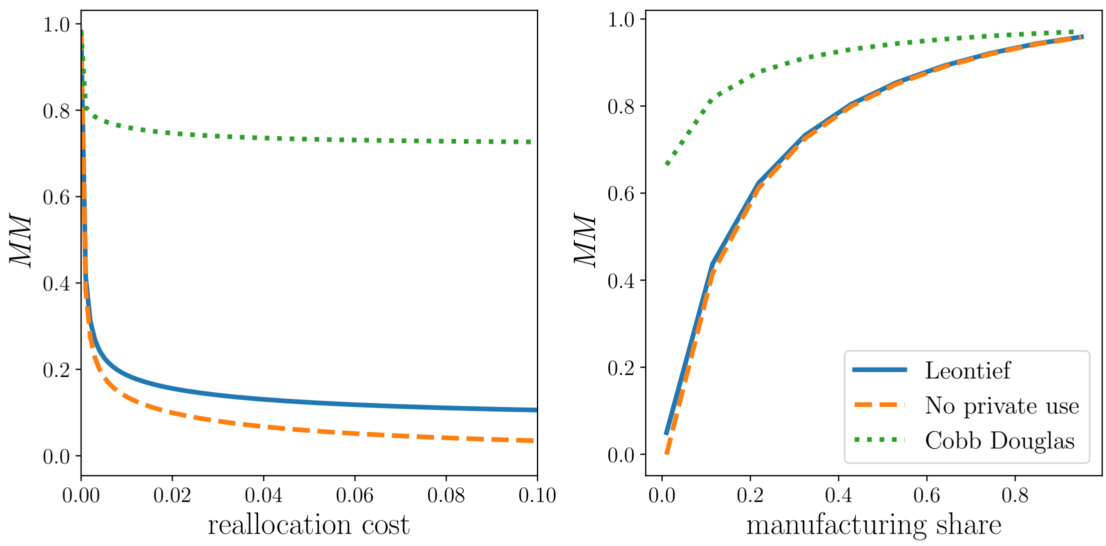
$$\epsilon^S \approx \frac{1}{1+\tilde{\phi}_{DM}} \cdot \frac{\xi_M}{\xi_D} + \frac{1}{1+\tilde{\phi}_{DS}} \cdot \frac{\xi_S}{\xi_D}. \tag{5.3}$$

That is, the supply elasticity depends now only on reallocation costs adjusted for a sector's size relative to the defense sector—to which we refer as "effective reallocation costs." For a given value of $\tilde{\phi}_{DM}$, the effective reallocation costs decline, and hence, the supply elasticity ϵ^S increases, as the size of *Manufacturing*, ξ_M, increases relative to *Defense*, ξ_D; and likewise for *Services*.

Figure 5 provides a graphical illustration in which circles represent sectors and circle size is proportional to sectoral value added, based on data from 2020 (left panel) and 1950 (right panel). The left panel showcases a salient feature of the post–Cold War economy: the dominance of services, which account for 75 percent of value added, compared with 4 percent for defense and 10 percent for manufacturing. The right panel depicts the Cold War economy, in which the manufacturing sector is almost three times larger.

It seems plausible to assume, as we do below, that capital reallocation between *Manufacturing* and *Defense* is less costly than between *Services* and *Defense*, as indicated by the thicker solid arrows in the figure. In this case, equation (5.3) implies that, holding pairwise reallocation costs $\tilde{\phi}$ fixed, *effective reallocation costs* vary with the relative sector size. They are higher in the post–Cold War econ-

Figure 6: Reallocation costs, industry structure and the *MM*



Notes: This figure plots how the on-impact *MM* depends on the reallocation cost (**left panel**), and manufacturing share (**right panel**). The **solid line** plots the case of Leontief production technology, the **dashed line** – without exports of military goods, and the **dotted line** Cobb-Douglas production function.

omy because of its industry structure—a notion that is central to interpreting the quantitative results below.

Figure 6 shows how the *MM* varies in a stylized three-sector economy as reallocation costs change (left panel) and as the manufacturing sector size varies (right panel). We consider three versions of the model that illustrate the effect of varying supply and demand elasticities: (i) Cobb–Douglas production ($\epsilon_D = 1$), shown by the dotted green line; (ii) Leontief production ($\epsilon_D = 0$), shown by the solid blue line; and (iii) Leontief production without private use of defense goods ($\epsilon^d = 0$), shown by the dashed orange line. Parameters follow the calibration described below, except that we aggregate to three sectors.[8]

In both panels, *MM* is measured on the vertical axis. In the left panel, reallocation costs are measured on the horizontal axis. When reallocation costs are zero, *MM* equals one in all three variants of the model. As reallocation costs increase, *MM* declines sharply, particularly under Leontief production. However, *MM* eventually plateaus. The reason is that even when reallocation becomes prohibitively costly, productive capacity in a sector can still be expanded

[8] Baseline sectoral shares correspond to the post-Cold War economy on Figure 5. The baseline reallocation cost between Manufacturing and defense: $\phi_{MD} = 0.045$ (median across disaggregated estimates for N-sector model below), and no reallocation is possible from Services.

through other channels, as we show below. Finally, eliminating private use of defense goods has little effect, since in our calibration the scope for crowding out is limited by the small share of private defense use.

In the right panel of Figure 6, we vary the combined share of *manufacturing* and *defense* in total value added along the horizontal axis—to which we refer as "manufacturing share." As this share increases, *MM* rises as well. The reason is that a larger industrial base lowers effective reallocation costs by raising the supply elasticity, see again expression (5.3). In the Leontief case, *MM* is essentially zero when an industrial base is absent. It is larger under Cobb–Douglas production, but in both cases *MM* increases with the industry share.

5.2 Model calibration

We calibrate the model to match key features of the post–Cold War economy, notably its production and investment networks, as well as the time-series evidence presented in Section 3. Importantly, to capture the input–output and investment networks, we distinguish $N = 64$ sectors: *Defense*, which is treated as a separate sector in the economic input–output accounts, and 63 other sectors. As our baseline, we adopt a Leontief production function. This captures the limited short-run substitutability in manufacturing, consistent with the low elasticities estimated by Boehm et al. (2019) and Atalay (2017). It also ensures that the calibrated reallocation costs are cleanly identified from the sectoral price responses: as expression (5.2) illustrates for the simplified economy, under Leontief the supply elasticity depends solely on reallocation costs and sector size. As shown in Figure 6, the results are qualitatively robust to Cobb–Douglas production; further, Figure C.1 shows that Cobb-Douglas with sectoral labor supply also aligns quantitatively well.

Consumption shares, production shares, and input-output matrix. For each sector, we pin down the parameters governing consumption shares, primary input shares, capital shares, and input–output shares using data from the "Use Tables" of the Bureau of Economic Analysis (BEA) for the year 2023, thereby capturing sectoral heterogeneity along these dimensions. After removing scrap, non-comparable inputs, and non-defense government sectors, we are left with 64 sectors.

Specifically, we set b_i to match the share of sector i in personal consumption expenditures. Primary input shares θ_i are calculated as one minus the

Table 1: Sector Linkages of Defense Sector

A) Input–Output Links (Intermediate Inputs to the Defense Sector)

Code	Share	Description	Code	Share	Description
22	0.0172	Utilities	331	0.0012	Primary metals
23	0.0457	Construction	332	0.0365	Fabricated metal products
311FT	0.0137	Food & tobacco	333	0.0060	Machinery
313TT	0.0021	Textiles & textile products	334	0.0706	Computers & electronics
315AL	0.0004	Apparel & leather	335	0.0166	Electrical equipment
321	0.0001	Wood products	3361MV	0.0367	Motor vehicles & parts
322	0.0012	Paper	3364OT	0.2452	Other transport equipment
323	0.0036	Printing & related support	339	0.0083	Misc. manufacturing
324	0.0578	Petroleum & coal products	481	0.0233	Air transportation
325	0.0145	Chemicals	482	0.0006	Rail transportation
326	0.0149	Plastics & rubber	483	0.0071	Water transportation
327	0.0002	Nonmetallic minerals	484	0.0192	Truck transportation
HS	0.0189	Health services	487OS	0.0015	Scenic & support transport
ORE	0.2359	Other real estate	493	0.0004	Warehousing & storage
5411	0.0237	Legal services	512	0.0194	Motion pictures & sound
5415	0.0004	Computer systems & consulting	513	0.0080	Broadcasting & telecom
621	0.0092	Ambulatory health care	523	0.0023	Securities & investments
622	0.0136	Hospitals	524	0.0017	Insurance
623	0.0053	Nursing & residential care	624	0.0165	Social assistance

B) Investment Links (Investment Goods Purchased by the Defense Sector)

Code	Inv. share	Description	Code	Inv. share	Description
311FT	0.0003	Food & tobacco	3364OT	0.0889	Other transport equipment
321	0.0672	Wood products	337	0.2497	Furniture
333	0.0011	Machinery	487OS	0.0362	Scenic & support transport
334	0.0130	Computers & electronics	512	0.0157	Motion pictures & sound
335	0.0920	Electrical equipment	523	0.0613	Securities & investments
3361MV	0.0042	Motor vehicles & parts	524	0.3694	Insurance

Notes: Sectoral links between each origin industry and the defense sector, measured as a share of total output, for production (top panel) and investment goods (bottom panel). Top panel based on 2023 BEA IO tables; bottom table based on 2018 table compiled by Vom Lehn and Winberry (2022).

sectoral ratio of total intermediate inputs to total final output. To illustrate, and because it is the focus of our analysis, we report the values for *Defense* in the top panel of Table 1: the share of intermediate inputs this sector sources from the other sectors of the economy. Quantitatively, inputs from most sectors play a moderate role, with notable exceptions for other transport equipment, real estate, and computers & electronics. *Other transportation equipment* (BEA code 3364OT) aggregates aerospace, shipbuilding, and military vehicle manufacturing—industries that form the core of the defense supply chain. Capital shares α_i are defined as one minus the sectoral ratio of compensation of employees to value added. ω_{ij} is set to match the share of input j in the total intermediate inputs of sector i.

Investment network. We specify the parameters governing the private investment network using the tables constructed by Vom Lehn and Winberry (2022). Their data are reported at a more aggregated level—37 sectors—which we map to our 63 non-defense sectors using the correspondence provided in their dataset. We use the table for the year 2018 (the latest available) and set the own-sector investment share—which reflects maintenance and repair expenditures and ensures model invertibility—to 0.35 in all sectors but defense (Atalay, 2017). To calibrate the investment shares in the defense sector for which Vom Lehn and Winberry (2022) provide no data, we use the BEA input–output table for 2023, computing the flow of investment goods from each sector to the defense sector as the sum of national defense gross investment in equipment, intellectual property, and structures. The bottom panel of Table 1 shows that according to our data investment goods in the defense sector are produced based on inputs from 12 sectors only.

Reallocation costs. The model features N^2 sector pairs for which we must specify the parameter ϕ_{ij}, which governs reallocation costs. To reduce the degrees of freedom, we assume that the defense sector may expand only at the expense of manufacturing sectors. Hence, we impose restrictively high reallocation costs that effectively rule out capital reallocation across non-defense sectors and from non-manufacturing sectors to *Defense*. For the defense sector and the 19 sectors in *Manufacturing* (corresponding to BEA sectors beginning with digit 3), we allow for heterogeneous reallocation costs by permitting distinct values for all $\{\phi_{D,i}\}_{i \in G}$, where G denotes the set of 19 manufacturing sectors. To pin down these parameters, we simulate the model's response to a military buildup and target the estimate for the 1-year MM in the post–Cold War economy, shown in Figure 4 above as well as response of the relative prices in each of the 19 sub-sectors of *Manufacturing*.

Throughout the simulations, we keep the values of other parameters constant at conventional values that are independent of the network. Specifically, we assume that a model period corresponds to one year, set the depreciation rate to $\delta = 0.1$, and the discount factor to $\beta = 0.96$. We assume a unitary Frisch elasticity of labor supply ($\gamma = 1$). Within the Military sector, we equate private consumption of military goods with US arms exports, which have been rising from 0.3% of GDP during the Cold War to 0.8% in 2020.[9] For the spending

[9]The US state department issues yearly summaries of the dollar amount US arms transfers. The Cold War average is obtained from "Foreign military sales, foreign military construction

Table 2: Price responses and reallocation costs in manufacturing

BEA Code	Industry Description	Model	Data	$\phi_{D,i}$
327	Nonmetallic minerals	0.2627	0.4218	0.0002
3364OT	Other transportation equipment	0.2630	0.1996	0.0002
311FT	Food, beverage, and tobacco	0.1858	0.5939	0.0056
324	Petroleum and coal products	0.1499	0.1545	0.0176
313TT	Textiles and textile products	0.1130	0.1071	0.0247
331	Primary metals	0.1559	0.1586	0.0294
3361MV	Motor vehicles and parts	0.1205	0.3715	0.0324
335	Electrical equipment and appliances	0.1127	0.1074	0.0363
325	Chemicals	0.0480	0.0579	0.0400
326	Plastics and rubber	0.1039	0.1069	0.0455
323	Printing and related support	0.0750	0.3231	0.0707
315AL	Apparel and leather	0.0047	0.0061	0.1180
337	Furniture	-0.0159	-0.0128	0.3343
333	Machinery	0.0847	0.1199	1.0895
322	Paper products	0.0034	0.0078	1.2444
334	Computer and electronic products	0.0567	0.1053	2.8088
321	Wood products	-0.0591	-0.1252	3.3432
332	Fabricated metal products	0.0354	-0.5939	3.6095
339	Misc. manufacturing	-0.0677	-0.2499	19.8606

Notes: This table summarizes the calibration of heterogeneous reallocation cost parameters between defense and 19 manufacturing sectors. The Model and Data columns report the elasticities of sectoral prices to a defense spending shock produced by the model and our empirical estimates (averaged over 5 years).

process, we assume an AR(2) process and set $\rho_1 = 1.4$ and $\rho_2 = -0.5$ to match the estimated IRF of defense spending in Section 3.

Formally, we construct a measure for the distance between the data and the model *MM* in the first year after the shock:

$$\Delta_{MM} = MM_{t=1}^{model} - MM_{t=1}^{data}$$

and the vector of distances for manufacturing price responses over the first 5 years:

$$\boldsymbol{\Delta_p} = \boldsymbol{p}_{t=1...5}^{model} - \boldsymbol{p}_{t=1...5}^{data}.$$

We also construct a weighting matrix $W = diag\{\widehat{Var}(\boldsymbol{p}_{t=1...5}^{data})\}$ where $\widehat{Var}$ is the normalized variance of each sectoral price response ($\sum \widehat{Var}(\boldsymbol{p}_{t=1...5}^{data}) = 1$). The normalization ensures equal weights on the *MM* target and the sectoral price group of targets. We then pin down parameters by minimizing the following

sales and military assistance facts as of September 30, 1990."

criterion function:

$$f(\{\phi_{D,i}\}_{i\in G}) = \Delta_{MM}^2 + \boldsymbol{\Delta_p}' W^{-1} \boldsymbol{\Delta_p}. \tag{5.4}$$

Table 2 reports the estimated price responses as well as the resulting cost parameters. Notably, the lowest reallocation costs are estimated for *Other transportation equipment* that covers aerospace, shipbuilding, and military vehicle manufacturing. Reallocation costs are also relatively low for *Nonmetallic minerals* (cement, concrete, ceramics used in military construction), *Primary metals* (steel and aluminum), and *Electrical equipment* (motors, generators, switchgear)—all classic defense-adjacent inputs. At the other extreme, reallocation costs are highest for sectors such as *Miscellaneous manufacturing* (toys, sporting goods) and *Wood products*.

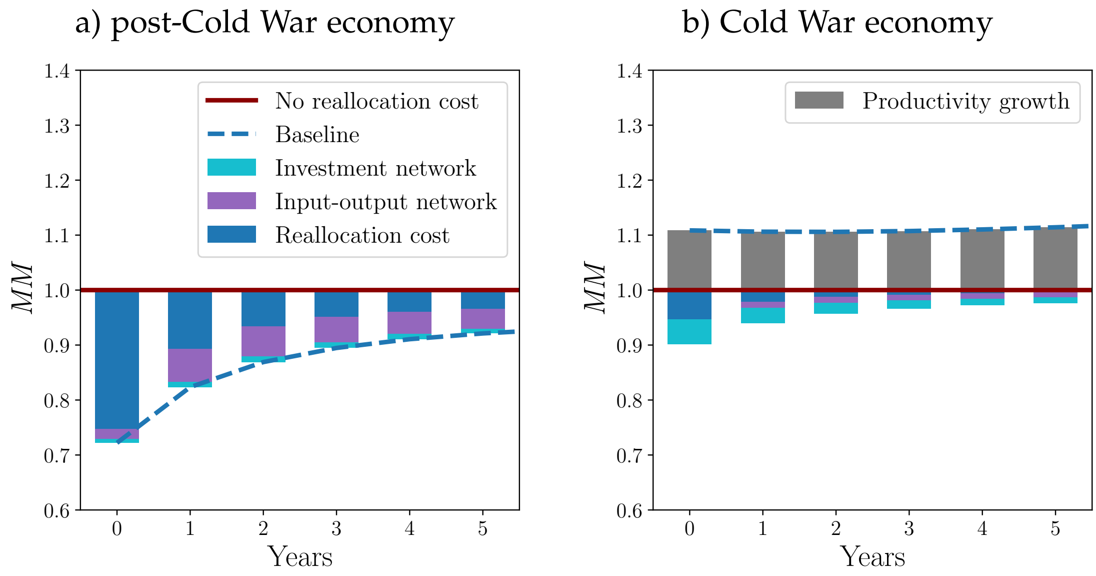
5.3 Results

Turning to the results, we first note that the calibrated model delivers accurate predictions for the *MM*. They are shown by the dashed blue lines in panel (a) of Figure 4 above. While the 1-year *MM* serves as a calibration target, the dynamics predicted by the model for the *MM* align well with the empirical evidence, too. Recall that it is initially well below unity and increases gradually over time. At the same time, the model also captures fairly well the heterogeneity of the price responses across sectors, as shown in Table 2.

Decomposition. We can now investigate what determines the military multiplier and, in particular, what causes it to be smaller than one. Figure 7 shows the results of a decomposition that quantifies the contributions of the different model features to the *MM*. The dashed blue line represents the baseline, reproducing the predictions of the calibrated model shown in Figure 4 above. To isolate the contribution of different model features to the *MM*, we report counterfactual outcomes obtained by switching off, in turn, the investment network, the input–output network, and finally a role for the industry structure by allowing for costless capital reallocation across sectors. As a natural benchmark, the solid red line shows the military multiplier in the one-sector economy, which is uniformly equal to unity.

Consider first the left panel of Figure 7 which shows the results for the post-Cold War economy which is our baseline. They are straightforward: in the absence of the investment network (light blue)—that is, when capital accumulation is based on a homogeneous investment good—the military multiplier is

Figure 7: Decomposition of *MM*



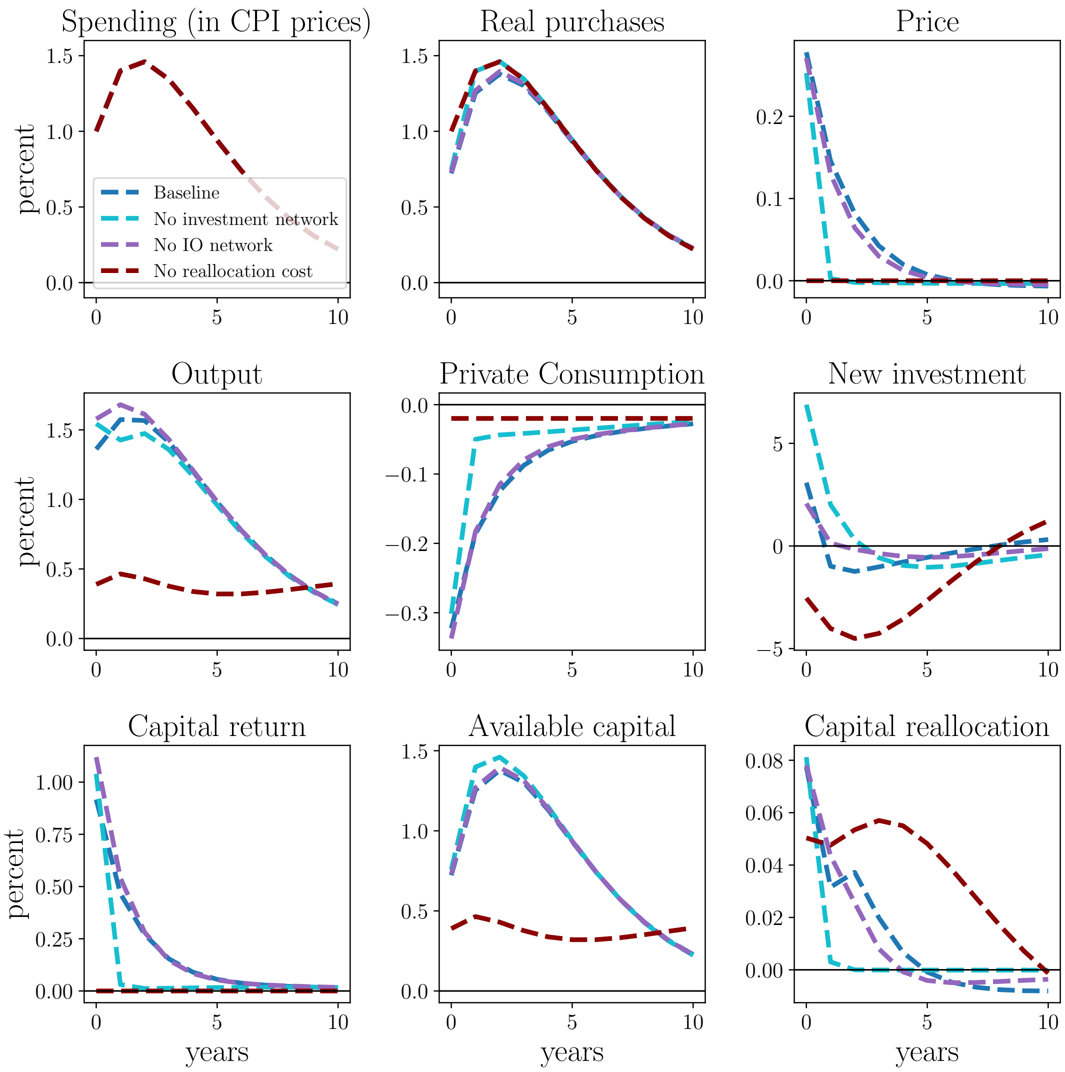
Notes: The figure plots the decomposition of the difference between the 1-sector-model MM and the N-sector-model *MM* to the effects of: defense-sector productivity (defense production efficiency), reallocation cost, input-output network, and investment network; both Cold-War and post-Cold War periods are considered.

somewhat larger, not only on impact but also in subsequent years.[10] The same holds for the input–output network (purple): without it, meaning that sectors rely on homogeneous intermediate inputs, the *MM* would be larger still. However, from a quantitative perspective, the most important feature is the industry structure, which—as discussed in Section 5.1—determines the *MM* by governing effective reallocation costs via the price elasticity of supply of defense goods, see again expression (5.3) above. To put this in perspective: removing reallocation costs closes the entire gap between the baseline *MM* of about 0.7 and the frictionless value of 1.0, whereas removing only the investment or the input–output network each closes only a modest fraction of this gap. And indeed, in Figure 7, when reallocation costs are switched off, the *MM* is unity, just as in the one-sector economy.

Figure 8 further zooms in on the underlying mechanism. It shows the impulse responses of selected variables to the military buildup, again contrasting the baseline responses with the same counterfactuals that we consider in Figure 7 above. The process for spending, measured in output units, is shown in the

[10]Specifically, we assume that in the absence of an investment (input-output) network, each sector uses the same investment (intermediate) good identical to the final consumption good.

Figure 8: Dynamic adjustment in the defense sector



Notes: Impulse responses to military buildup in baseline model (solid blue lines) calibrated to post-Cold-War economy.

upper left panel: it displays a hump-shaped pattern by virtue of the AR(2) assumption. Real purchases increase as well (upper-middle panel), but to a lesser extent because the relative price of defense goods rises (upper-right panel); the *MM* measures the wedge that the relative price drives between spending in output terms and in real terms. The price response differs between the baseline (blue solid line) and the counterfactuals. Importantly, if there are no reallocation costs (red line), prices do not increase and real purchases increase one-for-one with spending.

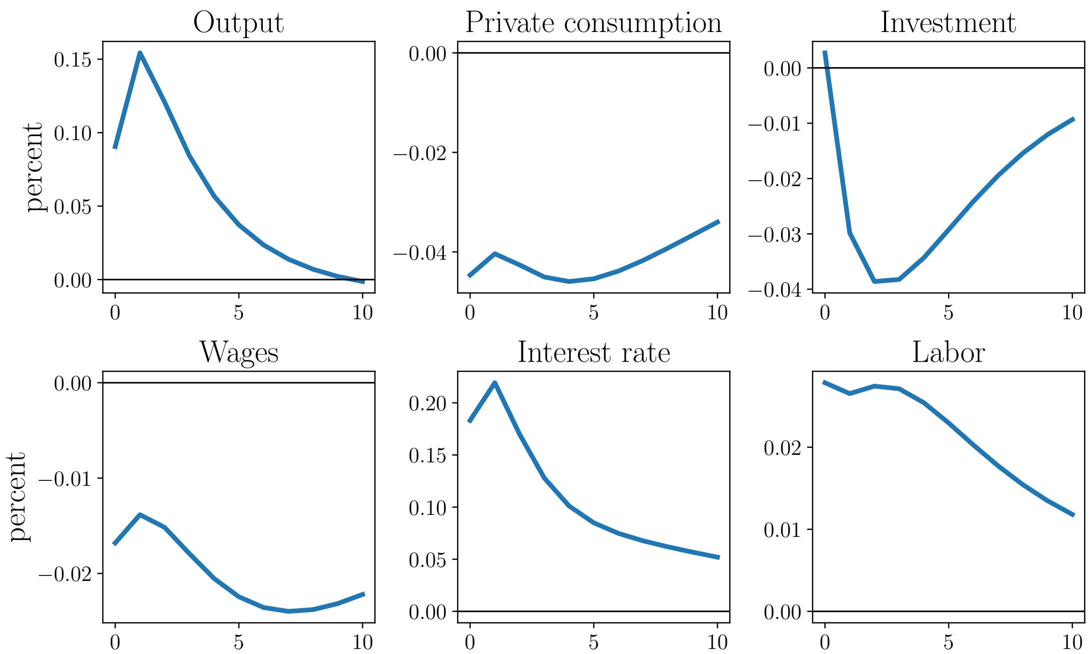
The left panel in the middle row shows the response of defense-sector output, which is not only purchased by the government but also used as an intermediate and investment good, as well as—though to a lesser extent—by the private sector. Here again, we observe large differences across scenarios. Output increases by less in all counterfactuals, particularly in the absence of reallocation costs. The middle panel shows the response of private consumption of defense goods: there is crowding out in all cases except when there are no reallocation costs. In that scenario, investment in the defense sector even declines—in contrast to the baseline and the other scenarios (right-middle panel).

To understand these dynamics, it is helpful to consider the adjustment paths of the variables shown in the bottom panels. In all cases, the return to capital in the defense sector increases, except when there are no reallocation costs. The intuition is straightforward: capital is reallocated to arbitrage away return differentials across sectors. This process turns out to be rather persistent (bottom-middle panel), precisely because the capital stock in the defense sector is initially drawn down. This effect is offset by prolonged reallocation dynamics, as shown in the bottom-right panel. Overall, the panels in Figure 8 underscore that reallocation costs dominate the adjustment dynamics from a quantitative perspective. Switching them off has a stronger effect than shutting down the networks.

Figure 9 complements the defense-sector perspective by showing the response of key macroeconomic aggregates to the military buildup. Output rises on impact, driven by the increase in government demand. Aggregate consumption, however, declines—reflecting standard crowding out as resources are reallocated toward the defense sector. Aggregate investment rises only on impact and then falls as well, as higher returns in the defense sector draw capital away from other uses. The fiscal multiplier—defined as the output response per unit of spending—is below unity and smaller than the one-sector benchmark, consistent with the resource costs imposed by the sectoral reallocation. These aggregate dynamics underscore that the inefficiency captured by the *MM* has broader macroeconomic consequences: the price pressure generated by the buildup not only reduces the real procurement of military equipment but also depresses private economic activity — consistent with evidence of crowding out by Barro and Redlick (2011).

The Cold War economy. It is instructive to contrast the result for the post-Cold War economy with those for a version of the model calibrated to capture the key features of the Cold War economy. For this purpose, we alter the cali-

Figure 9: Aggregate impulse responses to military buildup

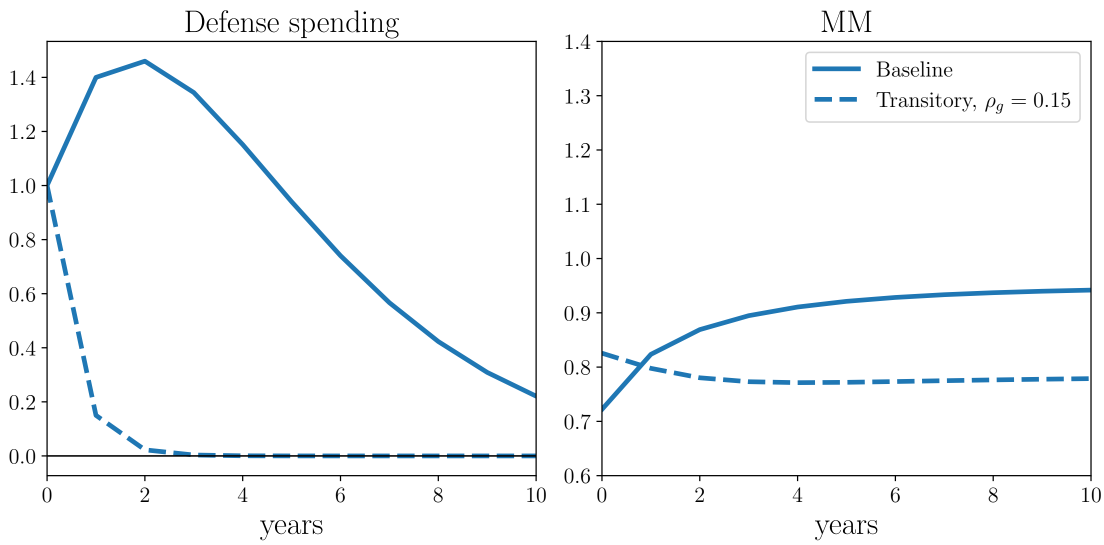


Notes: Impulse responses of macroeconomic aggregates to the military buildup in the baseline model calibrated to the post-Cold-War economy.

bration along three dimensions but, importantly, keep the reallocation cost parameters unchanged relative to the baseline (post-Cold War period). First, to pin down the parameters of the IO network we rely on the historical table for the year 1947. Because this table contains data for only 43 sectors, we set the corresponding shares in the missing sectors to zero. The 1947 table also lacks a separate defense sector and does not report information on labor income. In these instances, we therefore rely on the same defense-production shares and sectoral capital shares used in the post–Cold War calibration. Second, for the investment network we also use the 1947 table. Third, to allow the model to generate a *MM* larger than one, we allow for an exogenous improvement in defense production efficiency during military buildups. This captures well-documented channels through which military buildups can boost defense-sector productivity, including defense-related R&D spillovers and learning-by-doing in weapons production (Ilzetzki, 2024; Antolin-Diaz and Surico, 2025).

Based on the recalibrated model we repeat the decomposition of the *MM* and present the results in panel b) of Figure 7. The blue dashed line displays the model prediction which matches the evidence. It exceeds unity. Holding defense-sector productivity constant, the model predicts an impact *MM* of

Figure 10: Military multiplier and Buildup persistence



Notes: Figure plots the *MM* for persistent and transitory defense spending shocks. We consider the case of transitory shock (AR(1) with $\rho_g = 0.15$) against the baseline case.

roughly 0.9—substantially higher than in the post–Cold War calibration, yet still below the empirical response. Hence, we allow for an exogenous improvement in defense production efficiency which allows the model to generate a *MM* of about 1.1, in line with the estimates for this period. We then run counterfactuals to explore the role of the network and the industry structure. We find that these play a noticeably smaller role than in the post–Cold-War economy. Switching off either the investment or the input–output network raises the *MM* only modestly. Industry structure—operating through costly capital reallocation—creates the largest deviation from the frictionless benchmark, but even this deviation is only about half as large as in the post–Cold-War calibration—reflecting the larger industrial base in the Cold-War economy.

5.4 The persistence of military buildups

Military buildups are typically implemented at a specific point in time in response to geopolitical events unrelated to economic policy. In this sense, the *MM* is also beyond policymakers' control. However, our analysis highlights an important intertemporal dimension of military buildups, as expanding capacity in the defense sector takes time.

Against this background, we examine how *MM* varies with the persistence of the buildup. Figure 10 reports results under two alternative assumptions.

In both cases, we model defense spending as an AR(1) process. In the first scenario, we assume high persistence, setting the autocorrelation parameter to 0.96. In the second scenario, we assume low persistence and set the parameter to 0.15. The left panel of Figure 10 shows the implied path of defense spending: the solid line represents the persistent increase, while the dashed line depicts the more transitory increase.

The right panel shows the cumulative *MM* under both scenarios. With a persistent spending increase, the impact multiplier is slightly smaller, but the cumulative multiplier rises more strongly over time. This pattern reflects a twofold effect of persistent military spending. First, it generates an immediate increase in demand for military goods. Second, it signals that this higher demand will persist, which raises expected returns on military-related capital and leads to a sharper initial price response. Over time, higher investment expands capacity, making spending more efficient and amplifying the cumulative multiplier. This result carries a clear policy implication: credible, sustained defense commitments—such as binding long-term NATO spending targets—are more cost-effective than short-lived surges, as they allow the industrial base to expand in response to anticipated demand.

6 Conclusion

Military spending serves a different objective than stabilizing the business cycle. Whether these objectives—external security or geopolitical ends—can be met, depends on economic factors, among other things, how quickly and efficiently economic resources can be mobilized to meet a certain level of military capacity. In this paper, we put forward the notion of the military multiplier to account for this fact. Using U.S. time-series data and the defense news shocks of Ramey (2016), we estimate a short-run *MM* of approximately 0.7 in the post–Cold War period—meaning that roughly 30 cents of every additional dollar of defense spending are absorbed by rising prices rather than by additional equipment. In contrast, the *MM* exceeded unity during the Cold War, consistent with the observation that defense-sector prices fell in response to military buildups during that era.

Using a calibrated 64-sector business cycle model with production and investment networks, we show that the decline in the *MM* is driven almost entirely by the shrinking manufacturing base of the U.S. economy. While input–output and investment linkages play a role, their quantitative contribution is modest relative to the effect of effective reallocation costs, which rise as the industrial sectors from which the defense sector can draw capital become smaller. The model also reveals an important intertemporal dimension: credible, persistent buildups generate stronger capacity expansion over time, raising the cumulative *MM* substantially above its impact value.

Our findings carry direct implications for the current policy debate. In light of the Russian invasion of Ukraine and rising geopolitical tensions, many governments in Europe and the United States are embarking on substantial increases in defense spending. Our analysis cautions that the economic size of a country or alliance is not, by itself, a reliable indicator of how quickly military capabilities can be expanded. What matters is the structure of the economy—in particular, the size and composition of its industrial base—and the frictions that govern how resources are reallocated toward defense production. These considerations suggest that sustained, predictable commitments to defense spending are likely more effective than abrupt, short-lived increases.

Several avenues for future research are worth noting. First, a multi-country extension of our framework could yield important insights into how military procurement might be coordinated within entities such as the European Union, and how cross-country variation in the *MM* shapes the efficiency and strategic rationale of joint procurement. Second, as the nature of warfare evolves—for instance, with the growing role of drones, cyber capabilities, and artificial intelligence—the relevant sectors of the economy will shift, potentially altering the *MM* in ways that merit further study. Third, endogenizing the response of defense production efficiency to military buildups, rather than treating it as exogenous as we do for the Cold War calibration, would deepen our understanding of the long-run *MM*. We leave these extensions for future research.

References

Daron Acemoglu, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. The network origins of aggregate fluctuations. *Econometrica*, 80(5): 1977–2016, 2012.

Daron Acemoglu, Ufuk Akcigit, and William Kerr. Networks and the macroeconomy: An empirical exploration. *NBER Macroeconomics Annual*, 30(1):273–335, 2016.

Maxim Alekseev and Xinyue Lin. Export policy for dual-use goods. Manuscript, Working Paper, 2026.

Juan Antolin-Diaz and Paolo Surico. The long-run effects of government spending. *American Economic Review*, 115(7):2376–2413, 2025.

Enghin Atalay. How important are sectoral shocks? *American Economic Journal: Macroeconomics*, 9(4):254–280, 2017.

Adrien Auclert, Matthew Rognlie, and Ludwig Straub. The intertemporal keynesian cross. *Journal of Political Economy*, 132(12):4068–4121, 2024.

David Rezza Baqaee and Emmanuel Farhi. The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. *Econometrica*, 87(4):1155–1203, 2019.

Robert J. Barro. Output effects of government purchases. *Journal of Political Economy*, 89(6):1086–1121, 1981. doi: 10.1086/260102.

Robert J. Barro and Charles J. Redlick. Macroeconomic effects from government purchases and taxes. *Quarterly Journal of Economics*, 126(1):51–102, 2011. doi: 10.1093/qje/qjq002.

Marianne Baxter and Robert G. King. Fiscal policy in general equilibrium. *The American Economic Review*, 83(3):315–334, 1993.

Christoph E. Boehm, Aaron Flaaen, and Nitya Pandalai-Nayar. Input linkages and the transmission of shocks: Firm-level evidence from the 2011 Tōhoku earthquake. *Review of Economics and Statistics*, 101(1):60–75, 2019.

Hafedh Bouakez, Omar Rachedi, and Emiliano Santoro. The government spending multiplier in a multisector economy. *American Economic Journal: Macroeconomics*, 15(1):209–239, 2023.

Hafedh Bouakez, Omar Rachedi, and Emiliano Santoro. The sectoral origins of heterogeneous spending multipliers. *Journal of Public Economics*, 248:105404, 2025.

Olivier Cardi, Romain Restout, and Peter Claeys. Imperfect mobility of labor across sectors and fiscal transmission. *Journal of Economic Dynamics and Control*, 111:103815, 2020.

Lawrence J. Christiano, Martin Eichenbaum, and Sergio Rebelo. When is the government spending multiplier large? *Journal of Political Economy*, 119(1): 78–121, 2011. doi: 10.1086/659312.

Russell W. Cooper and Immo Schott. Capital reallocation and aggregate productivity. NBER Working Paper 19715, National Bureau of Economic Research, 2013.

Lydia Cox, Gernot J. Müller, Ernesto Pasten, Raphael Schoenle, and Michael Weber. Big *g*. *Journal of Political Economy*, 132(10):3260–3297, 2024.

Michael B. Devereux, Karine Gente, and Changhua Yu. Production networks and international fiscal spillovers. *The Economic Journal*, 133(653):1871–1900, 2023.

Andrea L. Eisfeldt and Adriano A. Rampini. Capital reallocation and liquidity. *Journal of Monetary Economics*, 53(3):369–399, 2006.

Andrea L. Eisfeldt and Adriano A. Rampini. New or used? investment with credit constraints. *Journal of Monetary Economics*, 54(8):2656–2681, 2007.

Jonathan-Julian Federle, Dominic Rohner, and Moritz Schularick. Who wins wars? Kiel Working Paper 2280, Kiel Institute for the World Economy (IfW Kiel), 2025. URL https://ideas.repec.org/p/zbw/ifwkwp/310329.html.

Francesco Ferrante, Sebastian Graves, and Matteo Iacoviello. The inflationary effects of sectoral reallocation. *Journal of Monetary Economics*, 140:S64–S81, 2023.

Joel P. Flynn, Christina Patterson, and John Sturm. Fiscal policy in a networked economy. Nber working paper, National Bureau of Economic Research, 2022.

Andrew T. Foerster, Pierre-Daniel G. Sarte, and Mark W. Watson. Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy*, 119(1):1–38, 2011.

Benjamin O. Fordham. The political and economic sources of inflation in the American military budget. *Journal of Conflict Resolution*, 47(5):574–593, 2003.

Michael Horvath. Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics*, 45(1):69–106, 2000.

Etan Ilzetzki. Guns and growth: The economic consequences of defense buildups. Kiel Report 2, Kiel Institute for the World Economy (IfW Kiel), 2025.

Ethan Ilzetzki. Learning by necessity: Government demand, capacity constraints, and productivity growth. *American Economic Review*, 114(8):2436–2471, 2024.

Stephan Jensen, Daniel Sleat, Christopher Wright, Tennyson Dearing, Yuliia Shevchuk, and Jacob Delmore. A secure future: The price of peace in ukraine and europe. Report, Tony Blair Institute for Global Change, 2025.

John Maynard Keynes. *The General Theory of Employment, Interest and Money*. Harcourt Brace, 1936.

Arvind Krishnamurthy and Annette Vissing-Jorgensen. The aggregate demand for treasury debt. *Journal of Political Economy*, 120(2):233–267, 2012.

Andrea Lanteri and Adriano A. Rampini. Constrained-efficient capital reallocation. *American Economic Review*, 113(2):354–395, 2023.

John B. Jr. Long and Charles I. Plosser. Real business cycles. *Journal of Political Economy*, 91(1):39–69, 1983.

Johannes Marzian and Christoph Trebesch. Guns and butter: The fiscal consequences of rearmament and war. Kiel Working Papers 2310, Kiel Institute for the World Economy, 2026.

Andrew Mountford and Harald Uhlig. What are the effects of fiscal policy shocks? *Journal of Applied Econometrics*, 24(6):960–992, 2009. doi: 10.1002/jae.1079.

Christopher J. Nekarda and Valerie A. Ramey. Industry evidence on the effects of government spending. *American Economic Journal: Macroeconomics*, 3(1):36–59, January 2011. doi: 10.1257/mac.3.1.36. URL https://www.aeaweb.org/articles?id=10.1257/mac.3.1.36.

Christopher J. Nekarda and Valerie A. Ramey. The cyclical behavior of the price-cost markup. *Journal of Money, Credit and Banking*, 52(S2):319–353, December 2020. doi: 10.1111/jmcb.12755.

Christopher Phelan and Alberto Trejos. The aggregate effects of sectoral reallocations. *Journal of Monetary Economics*, 45(2):249–268, 2000.

Christian Proebsting. Market segmentation and spending multipliers. *Journal of Monetary Economics*, 128:1–19, 2022.

Valerie A. Ramey. Identifying government spending shocks: It's all in the timing. *The Quarterly Journal of Economics*, 126(1):1–50, 2011.

Valerie A Ramey. Defense news shocks, 1889–2015: Estimates based on news sources. *Unpublished paper, University of California, San Diego*, 2016.

Valerie A. Ramey. Ten years after the financial crisis: What have we learned from the renaissance in fiscal research? *Journal of Economic Perspectives*, 33(2): 89–114, 2019.

Valerie A. Ramey and Matthew D. Shapiro. Costly capital reallocation and the effects of government spending. In *Carnegie–Rochester Conference Series on Public Policy*, volume 48, pages 145–194. Elsevier, 1998.

Valerie A. Ramey and Sarah Zubairy. Government spending multipliers in good times and in bad: Evidence from US historical data. *Journal of Political Economy*, 126(2):850–901, 2018.

Adriano A. Rampini. Financing durable assets. *American Economic Review*, 109 (2):664–701, 2019.

Reuters. Rising ammunition prices set back NATO efforts to boost security, official says, 2023. URL https://www.reuters.com/world/rising-ammunition-prices-set-back-nato-efforts-boost-security-official-2023-09-16/.

Vytautas Valaitis and Alessandro T. Villa. Optimal fiscal policy under endogenous disaster risk: How to avoid wars? Manuscript, Working Paper, 2025.

Christian Vom Lehn and Thomas Winberry. The investment network, sectoral comovement, and the changing US business cycle. *The Quarterly Journal of Economics*, 137(1):387–433, 2022.

Wei Wang. Capital reallocation: A tale of two frictions. *Scottish Journal of Political Economy*, 68(2):179–208, 2021.

Michael Woodford. Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35, 2011.

Appendix

A Data and Empirical Appendix

A.1 Time-series data

Unless otherwise noted, all series are available at quarterly frequency from 1947Q1 to 2018Q4 from the St. Louis FED - FRED database (mnemonics in parentheses).[11]

Producer price index of manufacturing goods. Producer price index by commodity: durable manufactured goods (WPUDUR0211, discontinued after 2018Q4) divided by the GDP deflator (GDPDEF). Monthly frequency aggregated to quarterly using the mean.

Defense expenditure deflator (national defense price index). Price index (implicit price deflator) for Government consumption expenditures and gross investment: Federal: National defense (B824RG3Q086SBEA; BEA NIPA Table 1.1.9), divided by the GDP deflator (GDPDEF). Quarterly frequency. BEA NIPA Handbook, Chapter 9 (Table 9.B), describes how the deflator is constructed from a weighted set of asset-specific price indexes for defense structures, equipment, and intellectual property products (e.g., RD), drawing on sources such as BLS producer price indexes, construction cost indexes, and hedonic indexes for certain high-technology equipment. These indexes are designed to measure constant-quality price change, so the resulting real series reflects quantities of defense capital after adjusting for inflation and quality improvements.

Ammunition prices. Manufacturing producer price index by industry: ammunition, except small arms (PCU332993332993) divided by the GDP deflator (GDPDEF). Only available from 1985Q4. Monthly frequency aggregated to quarterly using the mean.

Government spending. Government consumption expenditures and gross investment (GCE) divided by the GDP deflator (GDPDEF).

[11] In the quarterly regressions, we use only data until 2018Q4 to have a consistent sample across all dependent variables. The constraining factor is the availability of the military spending shocks of Ramey (2016) that run until 2015Q4.

Output. Nominal GDP (GDP) divided by the GDP deflator (GDPDEF).

Investment. Gross private domestic investment (GPDI) divided by the GDP deflator (GDPDEF).

Consumption. Sum of personal consumption expenditures for nondurable goods (PCND), durable goods (PCDG) and services (PCESV) divided by the GDP deflator (GDPDEF).

Public debt. Market value of gross federal debt (MVGFD027MNFRBDAL) divided by the GDP deflator (GDPDEF).

Inflation. Log-difference of GDP deflator (GDPDEF).

Nominal interest rate. Quarterly average of the effective federal funds rate (FEDFUNDS) until December 2008 and Wu-Xia Shadow federal funds rate afterwards.

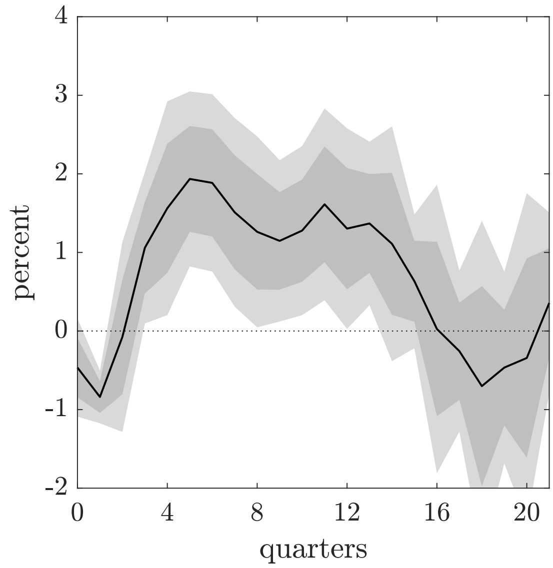
Real interest rate. Long-term rate on government bonds (yield on long-term US government securities (LTGOVTBD) until June 2000 and 20-year treasury constant maturity rate (GS20) afterwards) minus log-difference of GDP deflator (GDPDEF) (see Krishnamurthy and Vissing-Jorgensen, 2012).

Military spending shocks. Ramey (2016)-series of narratively-identified defense news shocks. Series available from 1947Q1 to 2015Q4 on Valerie Ramey's homepage (`https://econweb.ucsd.edu/~vramey/research.html`).

A.2 Additional empirical results

Figure A.1 shows the response of the ammunition producer price index to a military buildup shock in the post-Cold War period. The price response is substantially larger than that of the broad defense deflator reported in Section 3, consistent with ammunition being a narrowly defined and capacity-constrained defense input.

Figure A.1: Response of ammunition prices to the military buildup shock in the post-Cold War period



Notes: IRF based on narrative identification via military news series from Ramey (2016). Light (dark) gray areas are 90 (68) percent confidence bounds based on **?**-standard errors.

We extend the structural VAR framework of Nekarda and Ramey (2020) to examine the response of markups to government spending shocks across the Cold War and post–Cold War periods. Using the same specification, variable definitions, and trend adjustments as in the original study, we estimate impulse response functions for each subsample. As shown in Figure A.2, a positive spending shock leads to a modest, short-lived rise in markups during the Cold War, while the post–Cold War response is markedly weaker and statistically insignificant, suggesting that the markup adjustment mechanism has diminished over time.

Figure A.2: Markup response to government spending shocks (Cold War vs. post–Cold War)

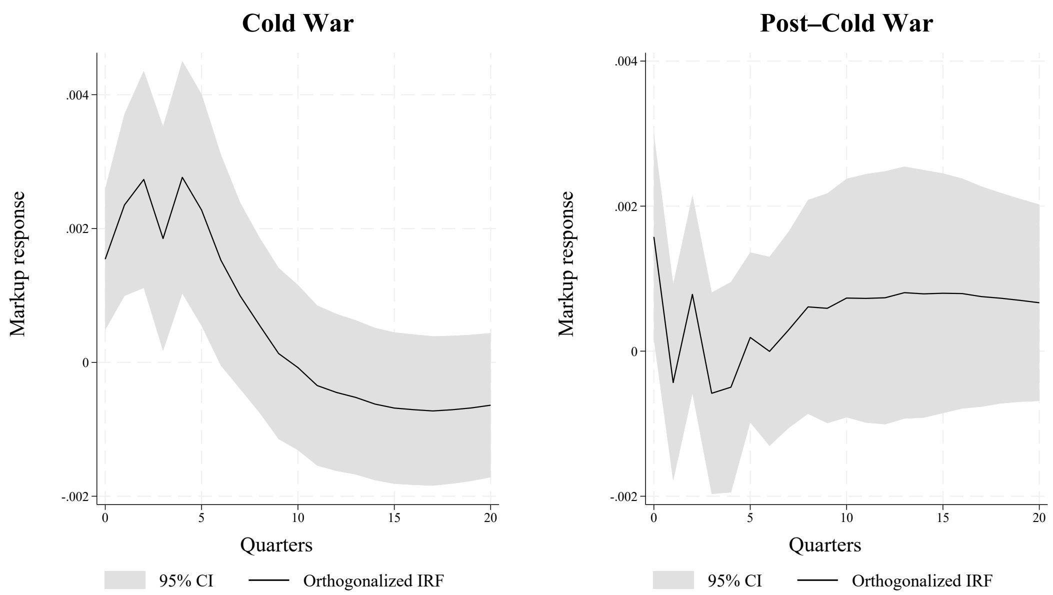
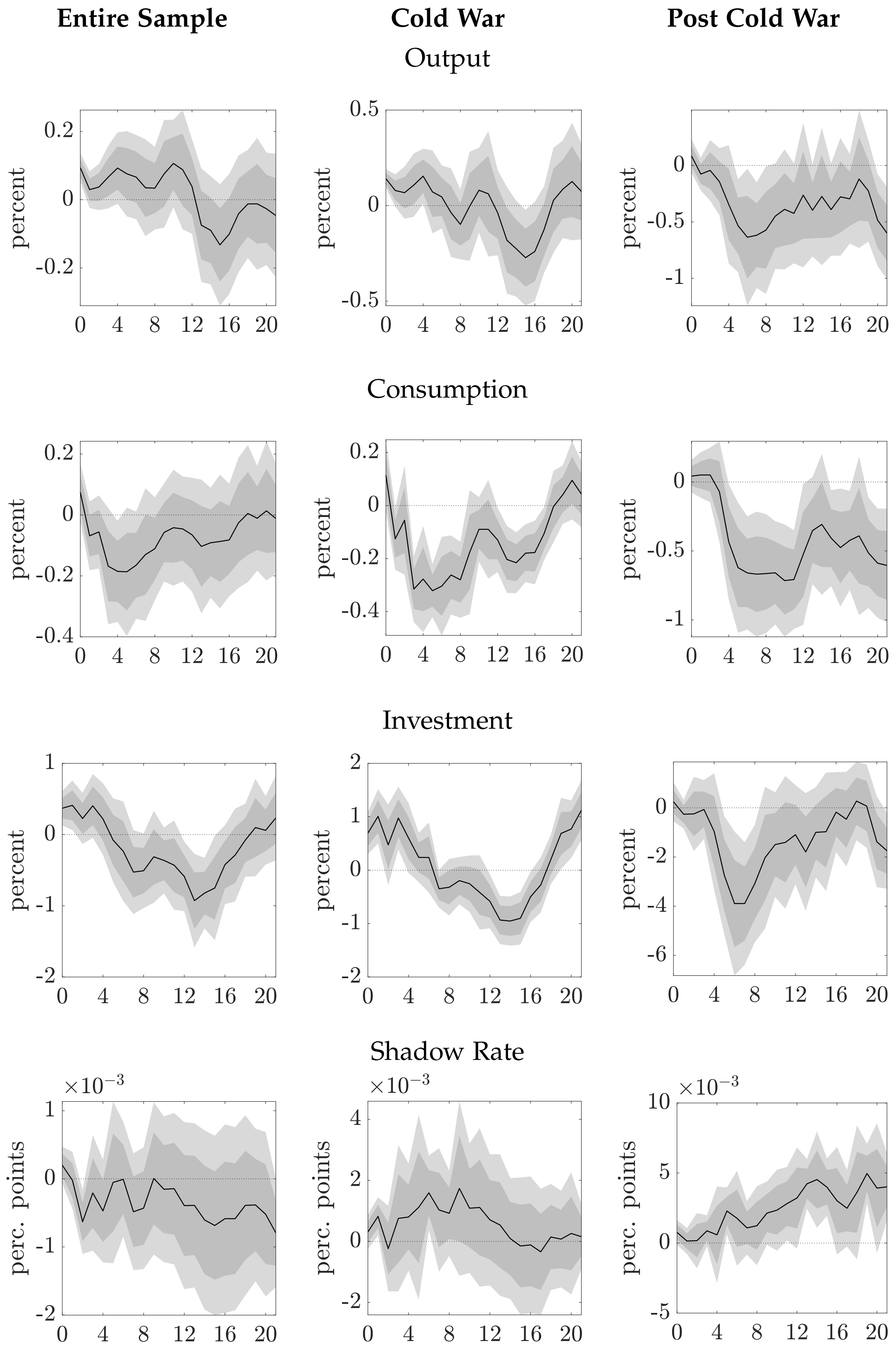
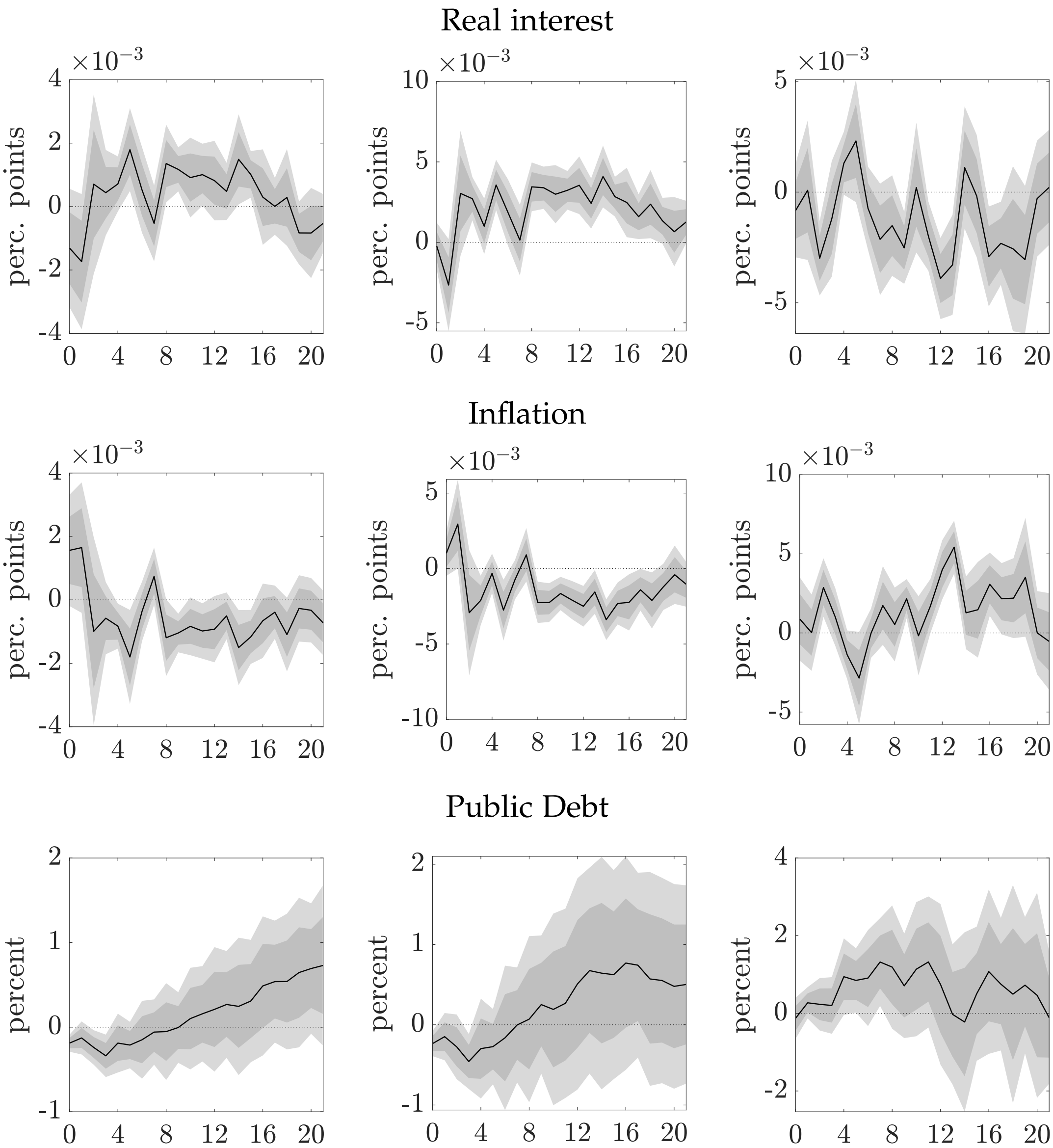


Figure A.3 reports impulse responses of additional macroeconomic aggregates—output, consumption, investment, interest rates, inflation, and public debt—to the military spending news shock, for the entire sample as well as the Cold War and post-Cold War subsamples separately.

Figure A.3: Empirical responses to fiscal expansion (US) - Additional variables





Notes: Impulse responses to a government spending shock identified via the military news series from Ramey (2016). Post-Cold War IRFs are scaled so that the peak government spending response equals that of the Cold War period. Light (dark) gray areas are 90 (68) percent confidence bounds based on **?**-standard errors.

B Model Appendix

B.1 Log-linearization

The model solution relies on log-linearization. We log-linearize the model around a steady state in which $\bar{P}_i = 1$ for all i (this normalization is arbitrary). Let us first introduce some notations.

Notation. Bar letter $\bar{X}$ denotes steady state value of X_t. Small letter denotes log-deviation from steady state $x_t = log(X_t) - log(\bar{X})$. Bold letters denote column vectors of sector-specific variables or parameters $\boldsymbol{x_t} = [x_{t,1}, ... \ x_{t,n}]'$; $\mathbf{1}$ denotes a column vector of ones. Matrix I_x denotes a diagonal matrix with vector $\boldsymbol{x}$ on the main diagonal; I denotes an identity matrix. Let the input-output matrix be denoted as W, such that $W(i,j) = \omega_{ij}$. The investment production matrix is denoted as W_λ, such that $W_\lambda(i,j) = \lambda_{ij}$.

Steady state. Since $\bar{P}_i = 1$ for all i, from 4.11 we have that $\bar{P}^I_i = 1$ for all i. From 4.1 we have that $\bar{Q} = \beta$. Then from 4.14 we have $\bar{r}_i = \frac{1}{\beta} - (1-\delta) = \bar{r}$ for all i. This implies that $\bar{P}^o_i = \bar{P}^o_j$ and that $\bar{R}_{ij} = 0$ for all i and j. Hence, there is *no reallocation in steady state*. Given the absence of reallocation, we have $\bar{I}_i = \delta \bar{K}_i = \delta \cdot \frac{\alpha_i \theta_i}{\bar{r}} \cdot \bar{P}_i \bar{Y}_i$ where the last equality follows from 4.6.

Now let us define the steady state sales shares (Domar weights) as $\xi_i = \frac{\bar{P}_i \bar{Y}_i}{\bar{P}\bar{C}} = \frac{\bar{Y}_i}{\bar{C}}$. Let us also define the steady state government spending shares as $\bar{g}_i = \frac{\bar{G}_i}{\bar{C}}$. Then, taking the resource constraint 4.21, multiplying by $\bar{P}_i$ and using equations 4.8 and 4.10, we obtain $\bar{P}_i \bar{Y}_i = \bar{P}_i \bar{C}_i + \sum_{j=1}^{N} (1-\theta_j)\omega_{ji} \bar{P}_j \bar{Y}_j + \sum_{j=1}^{N} \lambda_{ji} \frac{\delta}{\bar{r}} \alpha_j \theta_j \bar{P}_j \bar{Y}_j + \bar{P}_i \bar{G}_i$, which by dividing by $\bar{C}$ and rearranging yields the vector of sales shares $\boldsymbol{\xi} = [I - W'(I - I_\theta) - \frac{\delta}{\bar{r}} W'_\lambda I_\alpha I_\theta]^{-1} \cdot (\boldsymbol{b} + \bar{\boldsymbol{g}})$ where $\boldsymbol{b}$ is vector of consumption share parameters, $\bar{\boldsymbol{g}}$ is vector of steady state government spending shares. Then, the steady state labor shares are $\frac{\bar{W}\bar{L}_i}{\bar{C}} = (1-\alpha_i)\theta_i \cdot \xi_i$ and investment shares are $\frac{\bar{I}_i}{\bar{C}} = \delta \cdot \frac{\alpha_i \theta_i}{\bar{r}} \cdot \xi_i$.

Next we proceed with log-linearization. Log-linear Euler equation and labor supply are

$$q_{t,t+1} = E_t[c_t - c_{t+1}] \qquad \text{(Euler equation)} \qquad \text{(B.1)}$$

$$\gamma l_t = w_t - c_t \qquad \text{(Labor supply)} \qquad \text{(B.2)}$$

Sectoral consumption demand and consumer price index are

$$\boldsymbol{p_t} + \boldsymbol{c_t} = c \cdot \mathbf{1} \qquad \text{(Sectoral consumption demand)} \tag{B.3}$$

$$\boldsymbol{b'}\boldsymbol{p_t} = 0 \qquad \text{(Consumer price index)} \tag{B.4}$$

Sectoral labor supply aggregation is

$$[\boldsymbol{\xi'}(I - I_\alpha)I_\theta\mathbf{1}] \cdot l_t = \boldsymbol{\xi'}(I - I_\alpha)I_\theta \cdot \boldsymbol{l_t} \quad \text{(Sectoral labor aggregation)} \tag{B.5}$$

Labor demand and capital demand are

$$\boldsymbol{r_t} + \boldsymbol{\hat{k}_t} = \boldsymbol{p_t} + \boldsymbol{y_t} \qquad \text{(Capital demand)} \tag{B.6}$$

$$w_t\mathbf{1} + \boldsymbol{l_t} = \boldsymbol{p_t} + \boldsymbol{y_t} \qquad \text{(Labor demand)} \tag{B.7}$$

Sectoral prices (marginal cost) are

$$\boldsymbol{p_t} = -L\boldsymbol{a_t} + LI_\theta I_\alpha \boldsymbol{r_t} + L(I - I_\alpha)I_\theta \cdot \mathbf{1} \cdot w_t \quad \text{(Output prices)} \tag{B.8}$$

where $L = [I - W(I - I_\theta)]^{-1}$. Investment prices are

$$\boldsymbol{p_t^I} = W_\lambda \boldsymbol{p_t} \quad \text{(Investment prices)} \tag{B.9}$$

Capital reallocation and accumulation is

$$\boldsymbol{\hat{k}_t} = \boldsymbol{k_{t-1}} + \bar{r}[I_\alpha I_\theta I_\xi]^{-1}\boldsymbol{k_t^r} \qquad \text{(Available capital)} \tag{B.10}$$

$$\boldsymbol{k_t} = (1-\delta)\boldsymbol{\hat{k}_t} + \delta \boldsymbol{i_t} \qquad \text{(Capital accumulation)} \tag{B.11}$$

where $k_{t,i}^r = \frac{R_{t,i}}{\bar{C}}$ is the reallocated capital as the share of steady state consumption. Investment price dynamics is

$$\boldsymbol{p_t^I} = E_t[q_{t,t+1} \cdot \mathbf{1} + (1 - \beta(1-\delta)) \cdot \boldsymbol{r_{t+1}} + \beta(1-\delta)\boldsymbol{p_{t+1}^I}] \tag{B.12}$$

Price of existing capital is

$$\boldsymbol{p_t^o} = \bar{r} \cdot \boldsymbol{r_t} + (1-\delta)\boldsymbol{p_t^I} \tag{B.13}$$

Log-linearizing sector-pair reallocation, we get $k_{t,ij}^r = \frac{\bar{C}^{-1}}{\phi_{ij}+\phi_{ji}} \cdot (p_{t,i}^o - p_{t,j}^o)$. Then, sectoral reallocation is $k_{t,i}^r = \sum_j k_{t,ij}^r = p_{t,i}^o \sum_j \tilde{\phi}_{ij} - \sum_j \tilde{\phi}_{ij} p_{t,j}^o$ where $\tilde{\phi}_{ij} = \frac{\bar{C}^{-1}}{\phi_{ij}+\phi_{ji}}$. Then the link between sectoral prices of existing capital and sectoral reallocation are

$$\boldsymbol{k_t^r} = (I_\phi - W_\phi) \cdot \boldsymbol{p_t^o} \quad \text{(Capital reallocation)} \tag{B.14}$$

where matrix W_ϕ is such that $W_\phi i, j = \tilde{\phi}_{ij} = \frac{\bar{C}^{-1}}{\phi_{ij}+\phi_{ji}}$ and $I_\phi = diag\{W_\phi \cdot \mathbf{1}\}$. Finally, the resource constraint is

$$[I - W'(I - I_\theta)] \cdot I_\xi \cdot (\boldsymbol{p_t} + \boldsymbol{y_t}) = I_b\mathbf{1} \cdot c_t + \frac{\delta}{\bar{r}} W_\lambda' I_\alpha I_\theta I_\xi \cdot (\boldsymbol{p_t^I} + \boldsymbol{i_t}) + I_g \cdot (\boldsymbol{p_t} + \boldsymbol{g_t}) \tag{B.15}$$

B.2 Model solution

B.2.1 Reduced model system

To solve the model using BK method, we first simplify it to reduce the number of variables. This yields a system consisting of equations[12]

$$W_\lambda \boldsymbol{p_t} = E_t[(c_t - c_{t+1}) \cdot \mathbf{1} + (1 - \beta(1-\delta)) \cdot \boldsymbol{r_{t+1}} + \beta(1-\delta) W_\lambda \boldsymbol{p_{t+1}}]$$
$$\boldsymbol{k_t} = (1-\delta)\boldsymbol{\hat{k}_t} + \delta \boldsymbol{i_t}$$
$$[I - W'(I - I_\theta)] \cdot I_\xi \cdot (\boldsymbol{r_t} + \boldsymbol{\hat{k}_t}) = I_b \mathbf{1} \cdot c_t + [\frac{\delta}{\bar{r}} W_\lambda' I_\alpha I_\theta I_\xi W_\lambda + I_g] \cdot \boldsymbol{p_t} + \frac{\delta}{\bar{r}} W_\lambda' I_\alpha I_\theta I_\xi \cdot \boldsymbol{i_t} + I_g \cdot \boldsymbol{g_t}$$
$$\boldsymbol{p_t} = -L\boldsymbol{a_t} + L I_\theta I_\alpha \boldsymbol{r_t} + L(I - I_\alpha) I_\theta \cdot \mathbf{1} \cdot w_t$$
$$\boldsymbol{\hat{k}_t} = \boldsymbol{k_{t-1}} + \bar{r}[I_\alpha I_\theta I_\xi]^{-1} \boldsymbol{k_t^r}$$
$$\boldsymbol{k_t^r} = (I_\phi - W_\phi) \cdot [\bar{r} \cdot \boldsymbol{r_t} + (1-\delta) W_\lambda \boldsymbol{p_t}]$$
$$w_t = \frac{1}{1+\gamma} \cdot c_t + \frac{\gamma}{1+\gamma} \cdot \mathbf{1}'(I - I_\alpha) I_\theta \cdot (\boldsymbol{r_t} + \boldsymbol{\hat{k}_t})$$
$$\boldsymbol{b}' \boldsymbol{p_t} = 0$$

The first two systems are dynamic equations (contain next period variables). The rest are static equations. This system is complemented by the dynamics of the exogenous variables $\boldsymbol{a_t}$ and $\boldsymbol{g_t}$[13]

$$\boldsymbol{a_t} = \rho_a \boldsymbol{a_{t-1}} + \boldsymbol{\epsilon_t^a}$$
$$\boldsymbol{g_t} = \rho_g \boldsymbol{g_{t-1}} + \boldsymbol{\epsilon_t^g}$$

where $\boldsymbol{\epsilon_t^a}$ and $\boldsymbol{\epsilon_t^g}$ are exogenous shocks. The variables in the reduced system are: $\boldsymbol{a_t}$, $\boldsymbol{g_t}$, $\boldsymbol{k_{t-1}}$, $\boldsymbol{r_t}$, $\boldsymbol{\hat{k}_t}$, $\boldsymbol{i_t}$, $\boldsymbol{k_t^r}$, $\boldsymbol{p_t}$, c_t, w_t.

B.2.2 Model solution algorithm

Let $\boldsymbol{x_t}$ be a vector of variables. The system can be written as $A^0 E_t \boldsymbol{x_{t+1}} = A^1 \boldsymbol{x_t}$. Variables in $\boldsymbol{x_t}$ can be partitioned into dynamic variables $\boldsymbol{x_t^d}$ and static variables $\boldsymbol{x_t^s}$, that is $\boldsymbol{x_t} = [\boldsymbol{x_t^d}; \boldsymbol{x_t^s}]$. For static variables we have $A_{21}^0 = 0$ and $A_{22}^0 = 0$. Then, we have two underlying systems

$$A_{11}^0 E_t \boldsymbol{x_{t+1}^d} + A_{12}^0 E_t \boldsymbol{x_{t+1}^s} = A_{11}^1 \boldsymbol{x_t^d} + A_{12}^1 \boldsymbol{x_t^s}$$
$$0 = A_{21}^1 \boldsymbol{x_t^d} + A_{22}^1 \boldsymbol{x_t^s}$$

[12] The investment matrix W_λ should be full rank to ensure one-to-one mapping.

[13] Alternative (more realistic) government spending process is $\boldsymbol{g_t} = \rho_g^1 \boldsymbol{g_{t-1}} + \rho_g^2 \boldsymbol{g_{t-2}} + \boldsymbol{\epsilon_t^g}$ gives the hump-shaped government spending; requires extending state variables with $\boldsymbol{g_t^l} = \boldsymbol{g_{t-1}}$

Then, static variables can be mapped from dynamic variables as $\boldsymbol{x_t^s} = -[A_{22}^1]^{-1}A_{21}^1\boldsymbol{x_t^d}$. Substituting for static variables yields the following system

$$(A_{11}^0 - A_{12}^0 \cdot [A_{22}^1]^{-1}A_{21}^1)E_t\boldsymbol{x_{t+1}^d} = (A_{11}^1 - A_{12}^1 \cdot [A_{22}^1]^{-1}A_{21}^1)\boldsymbol{x_t^d}$$

which yields the standard system for BK method

$$E_t\boldsymbol{x_{t+1}^d} = A\boldsymbol{x_t^d}$$

where $A = (A_{11}^0 - A_{12}^0 \cdot [A_{22}^1]^{-1}A_{21}^1)^{-1} \cdot (A_{11}^1 - A_{12}^1 \cdot [A_{22}^1]^{-1}A_{21}^1)$. The dynamic variables are then partitioned into the state variables $\boldsymbol{x_t^{d,s}}$ and jump variables $\boldsymbol{x_t^{d,j}}$, that is $\boldsymbol{x_t^d} = [\boldsymbol{x_t^{d,s}};\ \boldsymbol{x_t^{d,j}}]$. Then solution follows standard BK method, resulting in the solution

$$\begin{aligned}\boldsymbol{x_{t+1}^{d,s}} &= M\boldsymbol{x_t^{d,s}} + \boldsymbol{u_{t+1}}\\ \boldsymbol{x_t^{d,j}} &= G\boldsymbol{x_t^{d,s}}\end{aligned}$$

where $\boldsymbol{u_{t+1}}$ is a vector of exogenous shocks.

In our system $\boldsymbol{x_t^{d,s}} = [\boldsymbol{a_t};\ \boldsymbol{g_t};\ \boldsymbol{k_{t-1}}]$ and $\boldsymbol{x_t^{d,j}} = \boldsymbol{r_t}$. Other variables are static.

B.3 Theoretical appendix

Proposition 2 (Military good market)**.** *Consider a multi-sectoral efficient economy with Military sector. The elasticity of military demand is:*

$$\epsilon_M^D = \underbrace{\sum_{f\in F} v_f^M \cdot \epsilon_f^D}_{\text{private uses}}$$

where F is a set of all non-government (private) uses of military good; v_f^M is the share of use f in total military output; ϵ_f^D is demand elasticity of use f.

The elasticity of military good supply is:

$$\epsilon_M^S = \underbrace{\epsilon \cdot \frac{1-\alpha_M}{\alpha_M}}_{\text{factor substitution}} + \underbrace{\frac{1}{\alpha_M\xi_M} \cdot \sum_{j=1}^{N} \frac{\alpha_j\xi_j}{1+\tilde{\phi}_{Mj}} \cdot \epsilon_{Mj}^R}_{\text{capital realloc.}}$$

where ϵ is substitution elasticity between capital and other factors of production; $\tilde{\phi}_{Mj}$ is capital reallocation cost from sector j to military sector, ξ_j size of sector j by sales, α_j capital share in production of sector j, ϵ_{Mj}^R is price elasticity of old capital reallocation from sector j to military sector.

Proof. We prove that the stated expressions hold by derivation. To derive demand link we start with the sectoral resource constraint for military sector:

$$Y_{t,i} = C_{t,i} + \sum_{j=1}^{N} X_{t,ji} + \sum_{j=1}^{N} I_{t,ji} + G_{t,i}$$

with $i = M$. Denoting steady state use shares as v_f^M and log-linearizing around the steady state, we get:

$$y_t^M = v_C^M \cdot c_t^M + \sum_{j=1}^{N} v_{Mj}^X \cdot x_t^{jM} + \sum_{j=1}^{N} v_{Mj}^I \cdot i_t^{jM} + v_G^M \cdot g_t^M$$

Let the private demand for each use be given by $x_t^{jM} = -\epsilon_{jM,X}^D \cdot p_t^M$ (same for consumption and investment uses). Then, substituting these private demand functions into the resource constraints and rearranging, yields the demand for the military good:

$$y_t^M = -[\sum_{f \in F} v_f^M \cdot \epsilon_f^D] \cdot p_t^M + v_G^M \cdot g_t^M$$

where $F = \{C, ..X_j.., ..I_j..\}$ is a set of all private uses of military good: private consumption, intermediate goods, and investment goods. v_f^M is the share of use f of military good in total military output (ex. $v_C^M = \frac{C_M}{Y_M}$ is ratio of private consumption of military goods to its output); ϵ_f^D is demand elasticity of f-th use of military good. The elasticity of military demand is:

$$\epsilon_M^D = \sum_{f \in F} v_f^M \cdot \epsilon_f^D$$

Hence, elasticity ϵ_M^D increases with the size of the private market as given by private use shares v_f^M and demand elasticity of each private use ϵ_f^D.

To derive supply equation, let us assume the arbitrary elasticity of substitution between factors of production ϵ. Then, demand for military capital can be written as

$$\hat{k}_t^M = -\epsilon \cdot (r_t^M - p_t^M) + y_t^M$$

Under simplifying assumptions of no input-output network $L = I$ and inelastic labor supply without wealth effect $w_t = 0$, and using the marginal cost expression, we can write

$$p_t^M = \alpha_M \cdot r_t^M$$

Substituting for interest rate, we obtain link between capital and price:

$$\hat{k}_t^M = -\epsilon \cdot (\frac{1}{\alpha_M} - 1) \cdot p_t^M + y_t^M$$

We approximate the quadratic reallocation cost with a linear function, by taking the constant marginal cost at a point $R_{Mj,t}$: $\frac{\phi_{jM}}{2}R^2_{Mj,t} \approx \tilde{\phi}_{jM}R_{Mj,t}$. Under the simplifying assumption of full depreciation $k^M_{t-1} = 0$, we have the link between capital reallocation and available capital:

$$\hat{K}^M_t - K^M_0 = \sum R_{Mj,t} - \frac{\phi_{jM}}{2}R^2_{Mj,t} \approx \sum(1-\tilde{\phi}_{jM})\cdot R_{Mj,t} \approx \sum \frac{R_{Mj,t}}{1+\tilde{\phi}_{Mj}}$$

where last approximate relationship stems from the fact that for small x we have $1-x \approx \frac{1}{1+x}$ and setting $\tilde{\phi}_{jM} = \frac{\phi_{jM}R_{Mj,t}}{2}$ the reallocation cost of given reallocation size. Reallocation in other sectors can be written as: $R_{Mj,t} \approx \hat{K}^j_t - K_0$, that is reallocated capital from sector j to Military should be equal to the change of capital in j.Then, using log-linearization we get: $k^r_{Mj,t} = \alpha_j\xi_i\hat{k}^j_t$ where $\alpha_j\xi_i = \frac{K_i}{C}$ is capital share in consumption. Using this expression, we obtain:

$$\hat{k}^M_t \approx \frac{1}{\alpha_M\xi_M}\sum\frac{\alpha_j\xi_j}{1+\tilde{\phi}_{Mj}}\hat{k}_{t,j}$$

Finally, let the supply of capital for sector j be given by $\hat{k}_{j,t} = \epsilon^R_{Mj}\cdot p^M_t$. From these relationships we get the military good supply as:

$$y^M_t = \left[\underbrace{\epsilon\cdot\frac{1-\alpha_M}{\alpha_M}}_{\text{factor substitution}} + \underbrace{\frac{1}{\alpha_M\xi_M}\sum_{j=1}^{N}\frac{\alpha_j\xi_j}{1+\tilde{\phi}_{Mj}}\cdot\epsilon^R_{Mj}}_{\text{capital realloc.}}\right]\cdot p^M_t$$

where ϵ is substitution elasticity between capital and other factors of production, α_M is capital share in military output; $\tilde{\phi}_{Mj}$ is reallocation cost from sector j to military sector $\xi_j = \frac{Y_j}{C}$ sales of sector j normalized by total private consumption, α_j capital share in production of sector j, $\bar{r}$ is steady state interest rate, ϵ^R_{Mj} is price elasticity of old capital supply from sector j to military sector.

Elasticity of military good supply is:

$$\epsilon^S_M = \underbrace{\epsilon\cdot\frac{1-\alpha_M}{\alpha_M}}_{\text{factor substitution}} + \underbrace{\frac{1}{\alpha_M\xi_M}\cdot\sum_{j=1}^{N}\frac{\alpha_j\xi_j}{1+\tilde{\phi}_{Mj}}\cdot\epsilon^R_{Mj}}_{\text{capital realloc.}}$$

Hence, this elasticity increases in the substitutability of capital with other factors of production ϵ. Second, it decreases with reallocation costs from each sector $\tilde{\phi}_{Mj}$ and increasing with the amount of capital in each of these sectors $\alpha_j\xi_j = \frac{K_j}{C}$ (normalized by total consumption). Also it increases with the price elasticity of supply of used capital from each sector to military sector.

□

Special case. Consider a $N = 3$ sector model, with Military, Industry, and Services sectors. We can make the following assumptions: 1) production functions are Leontief, 2) no reallocation is possible between Industry and Services, 3) investment in Services and Manufacturing is produced from aggregate consumption good, 4) capital intensity is 1 in all sectors (only capital is used in production), 5) discount factor is $Q_{t,t+1} \approx \beta = 1$, 6) reallocation cost is paid only by non-Military sectors ($\phi_{Mj} = 0$ while $\phi_{jM} > 0$). Under these assumptions, we can further simplify the expression for supply elasticity.

The (relative) price of old capital in $j \in \{I, S\}$ is the same in both sectors and $P^O_{t,j} = 1$. Here we used the result from the proof, that $p^j_t = \alpha^j r^j_t$, and the fact that r^j_t are such that $P^I_{t,j} = r^j_t$ with $P^I_{t,j} = 1$ (as consumption good price is normalized to 1). The old military capital price is then equal to the military good price $P^O_{t,M} = r^M_t = P^M_t$. Then, the reallocation of capital from sector j is $R_{t,j} = \frac{1-P^M_t}{\phi_{jM}}$. Taking log-deviations of the steady state, we obtain $\hat{k}^j_t = p^M_t$, that is $\epsilon^R_{Mj} = 1$.

B.4 Derivation of cumulative military multiplier

Cumulative multiplier derivation: Let $\{X_t\}^h_t$ be path of gov. spending after the shock. Then, total spending over h periods is $\sum^h_{t=1} X_t \approx \bar{X} \sum^h_{t=1} x_t$, where x_t is a percentage deviation from pre-shock value $\bar{X}$. Similarly, military equipment produced during these h periods is $\sum^h_{t=1} G_t \approx \bar{G} \sum^h_{t=1} g_t$. Dividing cumulative spending by cumulative military output yields the cumulative multiplier: $MM = \frac{\sum^h_{t=1} G_t}{\sum^h_{t=1} X_t} \approx \frac{\sum^h_{t=1} g_t}{\sum^h_{t=1} x_t}$. We further have, using $X_t = P_t \cdot G_t$,

$$\sum_{j=0}^{h} x_{t+j} = \sum_{j=0}^{h} g_{t+j} + \sum_{j=0}^{h} p_{t+j}.$$

Thus, the cumulative military multiplier can be written as

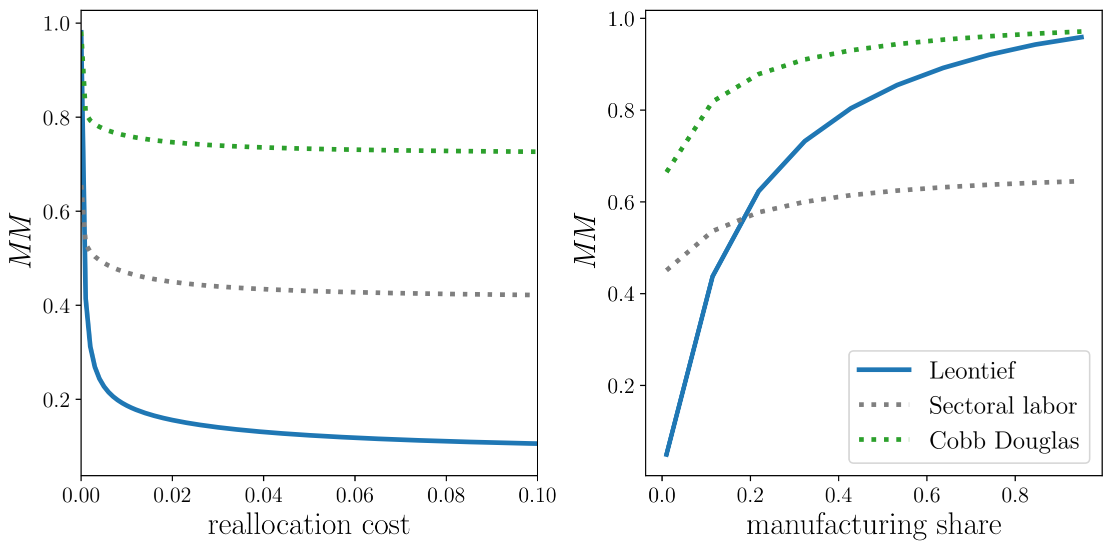
$$MM(h) = 1 - \frac{\sum^h_{j=0} p_{t+j}}{\sum^h_{j=0} x_{t+j}},$$

which we use to compute the cumulative multiplier directly from the data.

C Additional results

Figure C.1 explores how the on-impact MM varies with reallocation costs (left panel) and the manufacturing share (right panel) under three production technologies. The baseline uses Leontief production (solid line). The green dotted

Figure C.1: Reallocation costs, industry structure and the *MM*



Notes: **Solid line:** Leontief production. **Green dotted:** Cobb–Douglas. **Grey dotted:** Cobb–Douglas with sector-specific labor supply.

line replaces Leontief with Cobb–Douglas, allowing factor substitution within each sector. The grey dotted line further extends the Cobb–Douglas specification by introducing sector-specific labor markets: each sector i hires labor $L_{t,i}$ at a sector-specific wage $W_{t,i}$, and the aggregate labor supply function is replaced by a set of sector-specific labor supply functions $L_{t,i}^{\gamma} = W_{t,i}/C_t$, with no labor aggregation across sectors. This extension eliminates the pooled labor market, so that a military buildup cannot draw on workers from other sectors without bidding up their sector-specific wage.

Notably, the Cobb–Douglas specification with sector-specific labor supply is quantitatively close to the Leontief baseline across the entire range of reallocation costs and manufacturing shares. Once labor markets are segmented, within-sector factor substitution has limited bite, because expanding defense output still requires drawing resources from other sectors at increasing cost. What drives the *MM* is cross-sector capital mobility—the mechanism that the Leontief specification isolates cleanly (see expression (5.2)). The near-equivalence shown in Figure C.1 confirms that the results are driven by capital reallocation frictions rather than the specific assumption on within-sector factor substitution.