The Military Multiplier*

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Abstract

What determines the effectiveness of military buildups? We introduce the concept of the military multiplier: the percentage increase in military equipment an additional dollar buys. It varies with the costs of allocating resources to military production, depending, among other things, on the industrial structure and capital reallocation frictions. We show that the response of military-goods prices to military buildups is a sufficient statistic for the military multiplier and that it has declined over time in the US. Using a calibrated multi-sector business cycle model, we show this decline stems from the economy's structural shift toward the service sector.

Keywords: Military buildup, Government spending, Effectiveness, Sectors, Reallocation

JEL-Codes: H56, E62, E23, O41

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1 Introduction

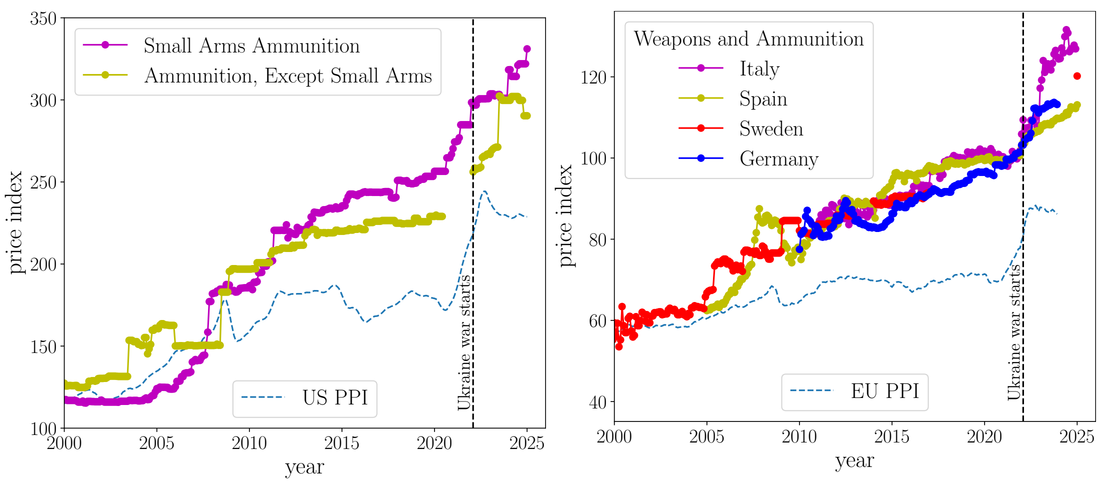
What determines the effectiveness of military buildups? Government spending—particularly military spending—is typically biased toward specific sectors of the economy (Cox et al., 2024). A large military buildup requires these sectors to expand, necessitating a reallocation of resources across the economy—a process that is both costly and time-consuming (Ramey and Shapiro, 1998). The costliness of this reallocation and the time horizon under consideration will influence how much military equipment an additional dollar of government spending can procure. Against this background, we define the *military multiplier* (*MM*) as the percentage increase in military equipment that can be obtained from an additional unit of output. In the short run, the *MM* will be less than 1 to the extent that the relative price of military equipment rises. It is distinct from the *fiscal multiplier*, which measures the percentage change in output resulting from an increase in government spending of one unit of output.

First, we show that the response of the relative prices of military equipment to military buildups is a sufficient statistic for computing the *MM* and document that shocks to US military spending do, in fact, induce significant changes in relative prices. The price of manufactured goods—which account for the lion's share of defense spending—increases significantly in response to the military spending news compiled by Ramey (2011, 2016), with a markedly stronger effect in the post-Cold War period than during the Cold War. Our estimates imply that the *MM* differs substantially across the two periods: in the short run, it is approximately 0.9 during the Cold War period but only about 0.4 in the post-Cold War period.

In partial equilibrium, the response of prices in the market for military goods simply reflects the price elasticities of supply and demand. However, in general equilibrium, these elasticities depend on how quickly the factors of production and demand are reallocated across sectors in response to a military buildup. We formalize this insight by developing a multi-sector business cycle model that accounts for networks in both, production and investment. We calibrate the model to match key features of the US economy in the Cold War and post-Cold War period, and we show how changes in industry structure account for the observed variation in the *MM*. The key difference lies in the relative size of the industrial and military sectors, which are significantly smaller in the latter period. Under these assumptions, the model's predictions for the magnitude of the *MM* align closely with the empirical evidence from both periods.

Figure 1: Weapon and ammunition prices in US and EU over time

(a) USA (b) EU countries



Notes: Sources: USA - BLS PPI data, EU - Eurostat PPI data.

In the model, the extent to which prices respond depends on how easily a sector's capacity can expand, which in turn hinges on the availability of factors of production—labor and capital. In our baseline specification, we assume that labor is mobile across sectors and that production follows a Leontief technology. As a result, the ease with which capital—the limiting factor—can be expanded within a sector is crucial. Capital can be accumulated through investment, but this process is both time-consuming and costly, as it requires inputs from a range of other sectors. To capture this, we explicitly model the investment network, following the approach of Vom Lehn and Winberry (2022).

Capital can also be reallocated across sectors, as in Ramey and Shapiro (1998), but reallocation is costly, too. These costs determine how quickly capital can adjust in response to a military buildup, and therefore influences the size of the *MM*. By varying adjustment costs, the model captures the full range of possible outcomes: a *MM* of zero when adjustment costs are infinite, a multiplier of one when adjustment costs are zero, and any value in between. For a given level of adjustment costs, the size of the military sector—as well as the sectors from which capital can be reallocated—plays a key role. The baseline version of the model includes three sectors: services, industry, and military production. We calibrate the model to match the industrial structure of both the Cold War and post-Cold War periods, and identify reallocation costs by targeting our empirical estimates for the *MM*.

We use the calibrated model to quantify the determinants of the *MM*, beyond the size of sectors and adjustment costs. In particular, we find that both, the investment and the production network play important roles. Assuming a small industrial base—similar to the post-Cold War economy—we find that if the investment network or the input-output network are more connected, the *MM* increases by approximately 50 percent. We also zoom in on the role of policy by computing the *MM* under alternative assumptions about the persistence of the military buildup. This persistence reflects, albeit in a stylized manner, whether the buildup is part of a credible long-term strategy. We identify a tradeoff: a more persistent buildup raises the *MM* in the long run, but reduces the impact multiplier, as prices initially respond more strongly to the anticipated sustained increase in demand.

In light of the Russian invasion of Ukraine and rising geopolitical tensions, many observers point to the economic strength of the European Union—whose economy is roughly ten times the size of Russia's—as evidence that expanding its defense capabilities and providing sufficient support for Ukraine should be relatively straightforward (see, for instance, Jensen et al., 2025). Indeed, recent work by Federle et al. (2025) highlights the critical role that economic resources play in determining the outcomes of wars.

However, this argument overlooks the underlying mechanics of the *MM*. Our model demonstrates that while the size of an economy and the availability of economic resources are important determinants of military capability, they are not sufficient on their own. Large economies may still lack the capacity to produce military equipment at the required pace. In fact, there is suggestive evidence that recent increases in military spending have, to a significant extent, been absorbed by rising prices. Figure 1 shows that, following the onset of the war in Ukraine in 2022, arms prices accelerated markedly in both Europe and the United States—outpacing the growth of the producer price index, see also, for example Reuters (2023).

We introduce the notion of the *MM* against the background of a large literature on the fiscal multiplier, which dates back to Keynes (1936), with modern treatments by Barro (1981), Woodford (2011), Auclert et al. (2024), and *many* others. The fiscal multiplier measures the percentage increase in output in response to an additional unit of government spending. The fundamental concern of this literature is how private expenditure (consumption, investment, and, in an open economy, net exports) responds to an increase in government spending, as this determines its effectiveness in stabilizing the economy—particularly when monetary policy is constrained by the zero lower bound (**?**). If private expenditure rises in response to additional government spending (is "crowded in"), the fiscal multiplier is greater than one; if it is crowded out, the fiscal multiplier is less than one.

In the context of our analysis, the response of private spending also matters for the *MM*. It is larger when non-government demand for military goods—whether from the domestic private sector or from foreign governments—is both substantial and price-elastic. In such cases, an increase in government demand tends to crowd out non-government demand, which in turn limits the rise in the price of military goods and reduces the need to meet additional demand through increased production. It follows directly that, all else being equal, an arms-exporting country will be characterized by a larger *MM*.

We note several caveats. First, as the nature of warfare evolves—such as with the replacement of fighter jets by drones—the sectors involved in the production of military equipment will also shift, at least to some extent. This does not invalidate the mechanics of the *MM*, but it may require a recalibration of the model. Second, we put forward a closed-economy model and abstract from the fact that military goods are sometimes imported. In the limiting case where the domestic economy is small relative to world markets, foreign procurement of military goods would not affect prices. However, reliance on foreign imports for military goods introduces its own set of risks. Third, our analysis focuses on the short run and abstracts from the longer-term effects of higher government spending on productivity (Antolin-Diaz and Surico, 2022; Ilzetzki, 2024). Fourth, our analysis is silent on the optimal level of defense spending, an issue that is studied by Valaitis and Villa (2025).[1] However, fundamental insights that follow from our analysis remain valid despite these caveats: economic strength alone is not a comprehensive measure of how easily military capabilities can be expanded; the structure of the industry is also of first-order importance.

The paper is structured as follows. In the remainder of the introduction, we place the paper in the context of the literature. In the next section, we formally develop the notion of the *MM* and establish some basic relationships. In Section 3 we estimate the impulse response of relative prices for manufacturing to military buildups. Section 4 presents the model. In Section 5 we consider a three-sector version of the model to illustrate our main point. A final section concludes.

[1] Alekseev and Lin (2025) study optimal trade taxes in the presence of a military externality due to dual-use goods.

Related literature. Our work relates to three strands of the literature. First and foremost, it builds on the seminal contribution by Ramey and Shapiro (1998), who highlight the importance of costly factor reallocation in response to a military buildup within a two-sector real business cycle (RBC) model. We extend this approach by incorporating costly capital reallocation into a state-of-the-art multi-sector RBC model with production and investment networks. Our focus also differs: we are concerned with the *MM*, a concept typically not explicitly analyzed in discussions of military buildups, see the recent survey Ilzetzki (2025), who notes that defense procurement should target quantities rather than nominal spending shares. More generally, however, recent work on fiscal policy has increasingly employed multi-sector models. Bouakez et al. (2023) and Acemoglu et al. (2016) show that sectoral linkages amplify and propagate government spending shocks, while Bouakez et al. (2022) emphasizes the importance of the sectoral origin of fiscal shocks. Devereux et al. (2023); Flynn et al. (2022) extend these insights to the regional and international trade. Ramey (2019) surveys the literature on government spending multipliers, highlighting the distinct effects of military versus non-military expenditures.

Second, there is the literature on the reallocation of the existing stock of capital (Eisfeldt and Rampini, 2006, 2007; Cooper and Schott, 2013; Rampini, 2019; Wang, 2021; Lanteri and Rampini, 2023). We show that capital reallocation plays a critical role in understanding the effectiveness of military buildups. We also contribute to the literature examining the macroeconomic effects of fiscal policy under factor immobility and sectoral heterogeneity. Cardi et al. (2020) and Proebsting (2022) show that costly labor mobility helps explain the macroeconomic responses to fiscal shocks. In contrast, we focus on the costly mobility of capital and explore its implications for the effectiveness of military buildups. More broadly, we relate to the literature on the aggregate effects of reallocation shocks under frictions. Phelan and Trejos (2000) and Ferrante et al. (2023) show that reallocation can have significant aggregate effects on output and inflation, highlighting the macroeconomic costs of shifting resources across sectors.

Finally, we relate to the literature on sectoral shock propagation in multi-sector real business cycle (RBC) economies (Horvath, 2000; Foerster et al., 2011; Atalay, 2017), as well as to studies emphasizing input-output linkages in production (Long and Plosser, 1983; Acemoglu et al., 2012; Baqaee and Farhi, 2019). We contribute to this literature by introducing a network of costly capital reallocation and demonstrating that it plays a crucial role in determining the effectiveness of sectoral government spending.

2 Military multiplier basics

Ultimately, the objective of military spending is to ensure that a specified quantity of military goods is available by a designated date. Against this background, we introduce a concept of *military multiplier* (*MM*) aiming to capture the ease with which the economy can convert the resources spent into the military equipment produced. Then, to set the stage, we take a partial equilibrium perspective and zoom in on the market for military goods.

2.1 Definition

To fix ideas, we consider an economy where the government only purchases military goods, G_t, from a specific sector while private consumption, C_t, and investment, I_t, have the same composition as GDP, Y_t, such that, with price indices appropriately defined, we have: $P_t^{PPI} Y_t = P_t^G G_t + P_t^{PPI} C_t + P_t^{PPI} I_t$. We deflate with P_t^{PPI} and define P_t as the relative price of government spending in units of output, which we use a numéraire good. Further, using hats to express the change of variables in terms of steady-state output, e.g., $\hat{g}_t = \frac{G_t - G}{Y}$; we write the percentage change of output as follows:

$$\hat{y}_t = \hat{x}_t + \hat{c}_t + \hat{i}_t, \tag{2.1}$$

where $\hat{x}_t \equiv \hat{p}_t + \hat{g}_t$ is the percentage change in military spending measured in units of output and $\hat{p}_t \equiv \frac{G}{Y} p_t$, assuming that $p = 1$ in steady state and letting letters without hats measure the percentage deviation of a variable from its steady-state value. Given this, we define two different multipliers.

- The **fiscal multiplier** is the percentage change of real output per percentage increase in government spending, measured in units of output:

$$M \equiv \frac{\hat{y}_t}{\hat{x}_t} = \frac{\hat{x}_t + \hat{c}_t + \hat{i}_t}{\hat{x}_t}.$$

- The **military multiplier** is the percentage change of real military spending per percentage increase in government spending, measured in units of output:

$$MM \equiv \frac{\hat{g}_t}{\hat{x}_t} = \frac{\hat{x}_t - \hat{p}_t}{\hat{x}_t} = 1 - \frac{\hat{p}_t}{\hat{x}_t} = \frac{\hat{g}_t}{\hat{g}_t + \hat{p}_t} = \frac{g_t}{g_t + p_t}. \tag{2.2}$$

Several remarks are in order. First, as we compute the military multiplier, we can disregard the response of non-military sectors, which is key for understanding the fiscal multiplier. Second, the military multiplier will differ from unity only as long as $p_t \neq 0$. Expression (2.2) also shows that the response of p_t is a sufficient statistic for backing out the military multiplier. Third, in one-sector models we have $p_t = 0$, such that M simplifies to $\frac{\hat{g}_t + \hat{c}_t + \hat{i}_t}{\hat{g}_t}$. Finally, in drawing a parallel between our concept of the military multiplier and the standard fiscal multiplier, we adopt the broader, modern interpretation of the latter, one that extends beyond the traditional Keynesian framework of fixed prices. The traditional Keynesian fiscal multiplier is a theoretical construct based on the assumption that, with fixed prices, private demand is entirely determined by current income, and output is fully driven by demand. In contrast, modern dynamic macroeconomic models typically assume incomplete price stickiness. As a result, private demand depends not only on current income but also on intertemporal prices—namely, interest rates—which adjust in response to government spending shocks. Thus, price adjustments are essential to determining the multiplier: relative prices for the military multiplier, and intertemporal prices for the fiscal multiplier.

Below, we also report the *cumulative military multiplier*, adapting the definition of Mountford and Uhlig (2009) or, equivalently the "present value multiplier," see also Ramey (2019). Formally, we compute the cumulative value of the response of real military spending over time divided by the cumulative value of the government spending response over time, measured in units of output, to the shock:[2]

$$\text{Cumulative } MM \text{ at lag k } = \frac{\sum_{j=0}^{k} g_j}{\sum_{j=0}^{k} (g_j + p_j)} = 1 - \frac{\sum_{j=0}^{k} p_j}{\sum_{j=0}^{k} x_j}. \tag{2.3}$$

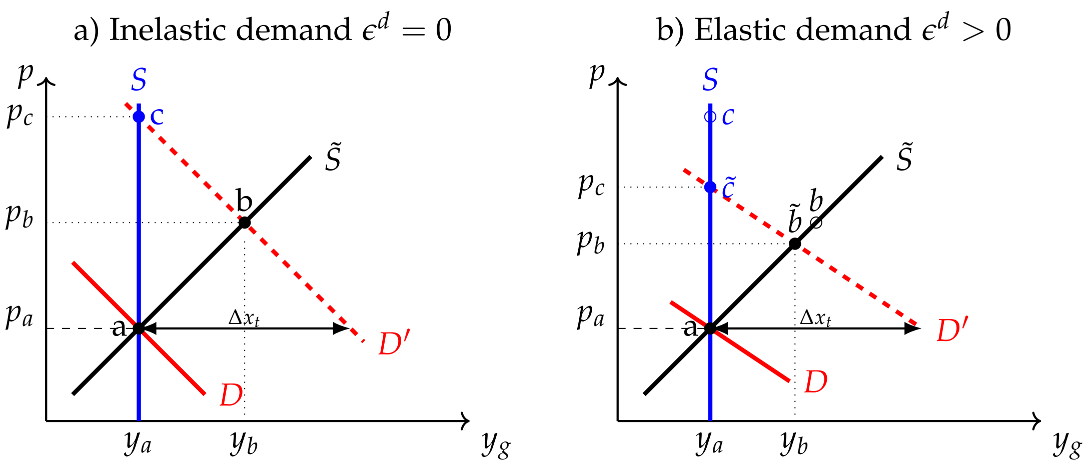
2.2 The market for military goods in partial equilibrium

The *MM*, as defined above, depends on how the price of military goods responds to the increase in spending triggered by the military buildup. From a partial equilibrium perspective on the market for military goods, the price response is determined solely by the price elasticities of both supply and demand. In what follows, we formalize this basic insight to set the stage for the subsequent sections.

In line with our full model presented below, we allow for the possibility that the government is not the sole buyer of military goods. There may also be private-sector demand—originating either domestically or abroad—with price elasticity denoted by ϵ^d. Using ϵ^s to denote the price elasticity of supply, we

[2]For simplicity, we do not discount spending that takes place later in time.

Figure 2: Equilibrium in the market for military goods



Note: Left (right) panel assumes inelastic (elastic) private-sector demand. Downward-sloping D-lines represent the transformed demand curve, see Equation (2.7). Supply curves S, see Equation (2.5), are shown for two cases: perfectly inelastic (vertical) and elastic (upward sloping).

write demand and supply in the market for military goods as follows:

$$y_{g,t} = -\epsilon^d \cdot p_t + g_t, \tag{2.4}$$

$$y_{g,t} = \epsilon^s \cdot p_t. \tag{2.5}$$

Substituting in (2.2), this implies for the MM:

$$MM = \left[1 + \frac{1}{\epsilon^d + \epsilon^s}\right]^{-1}, \tag{2.6}$$

that is, the MM is increasing in both, the elasticity of demand and supply: The more elastic both sides of the market, the less strong the increase in prices, and the more effective the military buildup.

It is instructive to rewrite the demand function (2.4) in such away that it depends on the government's purchases in terms of the numéraire good:

$$y_{g,t} = -(1 + \epsilon^d) \cdot p_t + x_t. \tag{2.7}$$

In this way, we capture the observation that military buildups typically specify a policy target in terms of the numéraire good x_t, for example, by specifying a certain spending target in percent of GDP. The extent to which military purchases change in real terms then depends on the price response. Figure 2 illustrates this graphically. Both panels of the figure show the equilibrium in the market for military goods, with prices measured along the vertical axis and quantities along the horizontal axis. The downward-sloping D-line represents equation (2.7) and shifts to the right as a result of the military buildup, specified in terms of the numéraire good, indicated by the dashed D′-line.

In the left panel we assume that private demand is perfectly inelastic and distinguish two scenarios for the price-elasticity of supply, indicated by the vertical blue line (inelastic supply) and the upward sloping black line (elastic supply). The implications are straightforward. If supply is inelastic, the outward shift in demand is fully absorbed by prices. There is no additional production of military goods (point c). The *MM* is zero. If instead, supply is elastic, both production and prices increase (point b).

In the right panel, we consider the case of a non-zero demand elasticity, meaning that as prices rise, private purchases of military goods are crowded out. As a result, the D-line is flatter, and as it shifts outward, prices increase less than in the case of inelastic demand—regardless of the supply scenario. Note that in this case, production (measured along the horizontal axis) also increases less, reflecting the decline in private-sector demand, which now makes room for the government's additional purchases. As emphasized above, what matters for the *MM* is the price response. This response is weaker the more elastic both demand and supply are.

Against this background, it is essential to understand the underlying determinants of both supply and demand in the market for military goods. To achieve this, we must shift from a partial to a general equilibrium perspective, since the shapes of both supply and demand ultimately depend on reallocation across sectors in response to the buildup. The multi-sector business cycle model in Section 4 provides such a perspective. In the next section, however, we first present evidence on how prices respond to military buildups.

3 Time-series evidence

As discussed above, the effectiveness of the military multiplier depends inversely on the movement of the relative prices of military-related goods: the more relative prices go up, the lower the multiplier. In what follows, we turn to time-series data for the US to assess whether the relevant prices respond significantly to military spending shocks. Specifically, we use manufacturing prices as a proxy for the price of military equipment, due to the lack of more granular data on military goods for longer time horizons. It is also worth noting the manufacturing sector is by far the largest sector when it comes to purchases by the Department of defense, receiving a share of 40 percent, see Table A.15 of the Online Appendix of Cox et al. (2024).

Hence, we proceed by looking into the response of relative manufacturing prices to military buildup shock in the US, distinguishing the Cold War and post-Cold War period. We run local projections as in Jordà (2005) and estimate the dynamic effects of military buildups using quarterly US data from *1947* to *2018*. Specifically, in the spirit of Ramey and Shapiro (1998), we run for each horizon h, a regression of the form:

$$y_{t+h} = \alpha_0 + \alpha_1 t + \sum_{i=1}^{8} b_i y_{t-i} + \sum_{i=0}^{8} c_i D_{t-i} + \varepsilon_t, \tag{3.1}$$

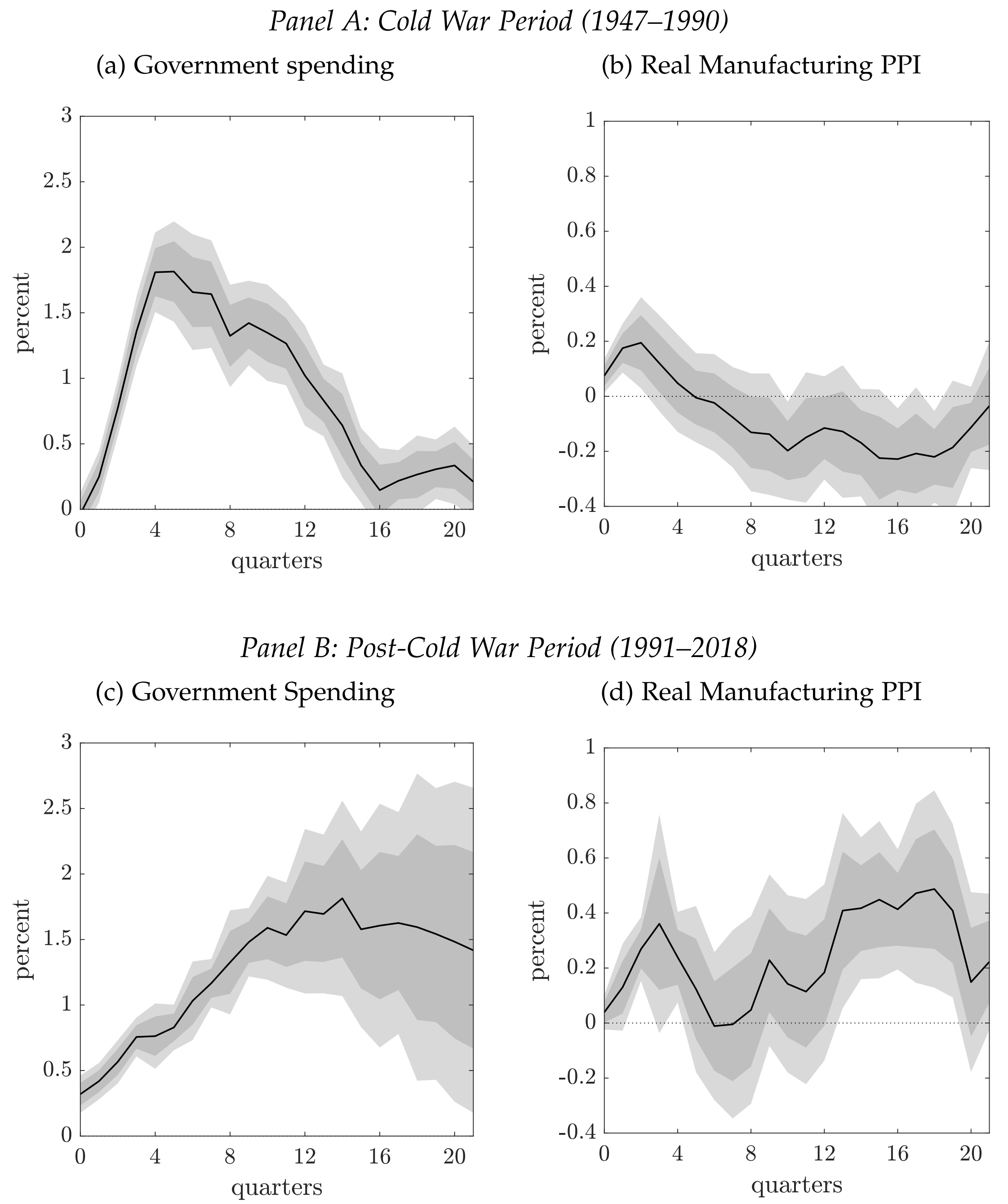
where y_{t+h} denotes the outcome variable of interest h periods ahead, and D_{t-i} is the military spending news shock from Ramey (2011). By estimating a distinct regression for each horizon, this approach allows us to trace out the impulse response without relying on a fully specified dynamic model.

We show results in Figure 3. The horizontal axis measures time in quarters, the vertical axis measures percentage deviation from the pre-shock level. Solid lines represent the point estimates while shaded areas indicate 68 and 90 percent confidence bounds. We split the sample into the Cold War period (1947–1990) and the post–Cold War period (1991–2018). The top panels of the Figure show the responses of real government spending and real manufacturing prices to the shock during the Cold War. Spending increases gradually and peaks about 4 quarters after the shock, remains high for an extended period and gradually reverts back to the pre-shock level. The real price of manufacturing goods increases for about four quarters, peaking at approximately 0.2%, before reverting quickly. This result is consistent with Ramey and Shapiro (1998), who also report estimates for the price of manufacturing to military news, using data through 1996 only.

The bottom panels of the same figure display different dynamics for the same variables, based on estimates for the post-Cold War period. In this case, the buildup of spending is more gradual but more persistent. We observe the peak only about 3 years after the shock. At the same time, the price response is much stronger and persistent: the real price of manufacturing goods increases permanently by about 0.4%.

Using the sufficient statistic established in the previous section, we can directly compute the cumulative military multiplier and assess how it has changed

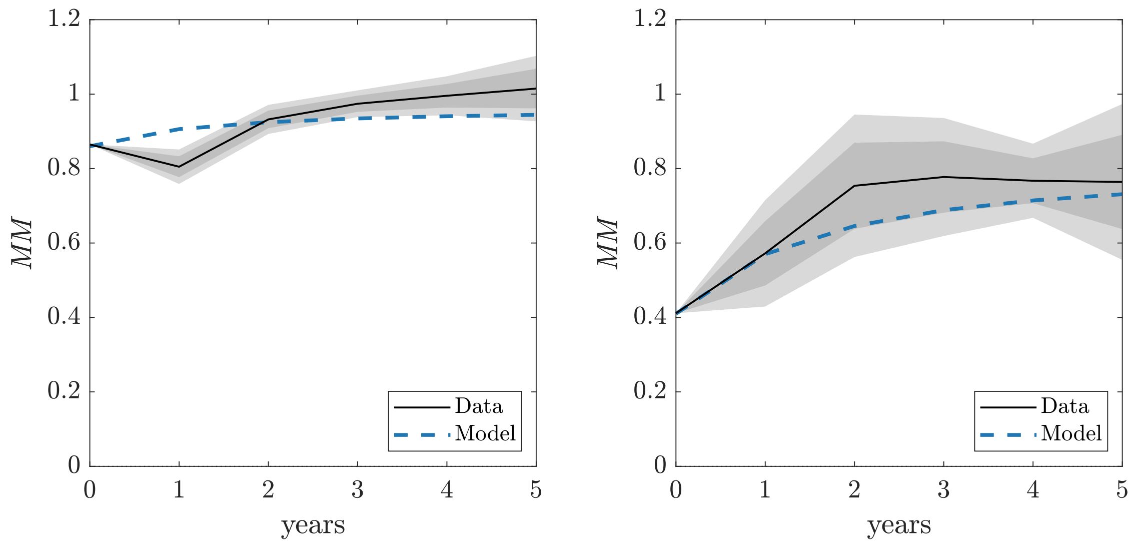
Figure 3: Response to military spending news



Notes: Quarterly response of government spending and manufacturing prices (deflated with GDP) to military spending news from Ramey (2016) during the Cold War (1947–1990) and Post-Cold War (1991–2018) periods. We normalize the peak response of government spending to be the same size across samples.

Figure 4: Cumulative military multiplier

(a) Cold War (b) Post-Cold War



Notes: Cumulative military multiplier over the first 5 years after the military buildup shock. Left panel: *MM* in Cold War Period (data) and in industrial economy model (large manufacturing sector). Right panel: *MM* in Post-Cold War Period (data) and service economy model (small manufacturing sector). Black solid lines represent the point estimates while shaded areas indicate 68 and 90 percent confidence bounds.

over time. Formally, we compute $MM(k) = 1 - \frac{\sum_i^k p_{t+i}}{\sum_i^k x_{t+i}}$ and show the results for both samples in Figure 4.[3] We find a military multiplier in the first year of the buildup of 0.86 in the Cold War, but only 0.41 in the post-Cold War period. The cumulative multiplier increases over time and actually becomes larger than 1.0 in the Cold War (left panel), suggesting that there were productivity gains in the production of military equipment. In contrast, the military multiplier only reaches a value of 0.76 after five years in the post-Cold War period.

Before rationalizing these findings with our model, we provide further evidence using more granular data. First, in the absence of historical data on arms prices, we use manufacturing prices as a proxy. Since manufacturing includes a much broader category than military goods, we expect the price response in this sector to be weaker than that of actual weapons prices. As a result, our es-

[3] We follow Ramey and Zubairy (2018) to obtain standard errors using a local projection instrumental variables (LP-IV) approach and estimate a regression of the form: $p_{t+h} = \alpha_0 + \alpha_1 t + b_i \sum_{i=1}^4 x_{t-i} + \sum_{i=0}^4 c_i X_{t-i} + \varepsilon_t$, where x_{t-i} are instrumented with the news spending shocks, and X_{t-i} is a vector of controls, including eight lags of p_t, x_t, as well as of the shock series.

timates likely overstate the military multiplier. Consistent with this notion, we find for the post-Cold War period—for which data on arms and ammunition prices are available—that arms prices are more responsive than manufacturing prices, see Appendix A.2.

The second caveat concerns the limited number of military news shocks in the post-Cold War sample, which leads to noisier estimates of the price response. The limited number of shocks raises the possibility that the observed response in manufacturing prices may reflect broader sectoral trends rather than military-specific effects. However, the strong movement in arms prices over the same periods suggests that the broader rise in manufacturing prices is indeed driven, at least in large part, by military developments.

4 A multi-sector economy

Our model is based on the multi-sector real business cycle model with input-output and investment networks of Vom Lehn and Winberry (2022). To study the effects of military buildups, we extend their framework in two dimensions. First, we allow for costly reallocation of capital across sectors. Second, we introduce sectoral government spending following Cox et al. (2024). These extensions allow us to model the short-run sectoral dynamics related to the military buildups.

4.1 Households

A representative household maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \frac{L_t^{1+\gamma}}{1+\gamma} \right]$$

subject to the budget constraint $C_t + Q_{t,t+1}B_{t+1} = B_t + W_t L_t + T_t$. Here β and γ are positive constants and E_0 is the expectation operator. C_t is consumption, L_t is hours worked, B_t is bond holdings, $Q_{t,t+1}$ is the bond price, W_t is the wage rate, T_t are transfers of profits from owning all firms in the economy and government taxes/transfers. Note that consumption price is normalized to one; all other prices (and wages) are expressed in relative terms. Ruling out Ponzi schemes, the first-order conditions are given by:

$$Q_{t,t+1} = \beta \cdot E_t \frac{C_t}{C_{t+1}}, \qquad \text{(Euler equation)} \qquad (4.1)$$

$$L_t^{\gamma} = \frac{W_t}{C_t}. \qquad \text{(Labor supply)} \qquad (4.2)$$

Sectoral consumption demand. Consumption C_t is an aggregate of sector-specific consumption goods: $C_t = \bar{b} \prod_{i=1}^{N} C_{t,i}^{b_i}$ where N is the number of sectors and $C_{t,i}$ is consumption of sector-i good. $\sum_{i=1}^{N} b_i = 1$ and $\bar{b} = [\prod_{i=1}^{N} b_i^{b_i}]^{-1}$ is a normalizing constant. Let $P_{t,i}$ be the price of sector-i good. Then, the sector-specific consumption demand and consumer price index are given by:

$$P_{t,i} C_{t,i} = b_i \cdot C_t, \qquad \text{(Sector } i \text{ consumption demand)} \qquad (4.3)$$

$$\prod_{i=1}^{N} P_{t,i}^{b_i} = 1, \qquad \text{(Consumer price index)} \qquad (4.4)$$

Sectoral labor supply. Total hours worked consists of labor supplied to each of N sectors, that is

$$L_t = \sum_{i=1}^{N} L_{t,i}, \quad \text{(Labor aggregation)} \qquad (4.5)$$

where $L_{t,i}$ labor supplied to sector i. Note that in the baseline model, labor is perfectly mobile across sectors. Additionally, we consider an alternative specification with sector-specific labor.

4.2 Production

Each sector consists of a set of identical perfectly competitive firms. Firms in sector i produce output $Y_{t,i}$ based on a sector-specific CRS production technology:

$$Y_{t,i} = F_i(A_{t,i}, \hat{K}_{t,i}, L_{t,i}, ..X_{t,ij}, ...)$$

For the Cobb-Douglas production function, we have:

$$Y_{t,i} = \bar{\omega} A_{t,i} \cdot \left(\hat{K}_{t,i}^{\alpha_i} L_{t,i}^{1-\alpha_i} \right)^{\theta_i} \cdot \left(\prod_{i=j}^{N} X_{t,ij}^{\omega_{ij}} \right)^{1-\theta_i},$$

where $\hat{K}_{t,i}$ is capital input, $L_{t,i}$ labor input, $X_{t,ij}$ sector-j output used as intermediate input in sector i; $A_{t,i}$ is sector-specific productivity.

$$r_{t,i}\hat{K}_{t,i} = \alpha_i\theta_i \cdot P_{t,i}Y_{t,i}, \qquad \text{(Sector } i \text{ capital demand)} \quad (4.6)$$

$$W_{t,i}L_{t,i} = (1-\alpha_i)\theta_i \cdot P_{t,i}Y_{t,i}, \qquad \text{(Sector } i \text{ labor demand)} \quad (4.7)$$

$$P_{t,j}X_{t,ij} = (1-\theta_i)\omega_{ij} \cdot P_{t,i}Y_{t,i}. \qquad \text{(Sector } i \text{ intermediate input demand)} \quad (4.8)$$

Given perfect competition, the sector-specific price is equal to marginal costs and given by:

$$P_{t,i} = \frac{1}{A_{t,i}} \cdot \left(r_{t,i}^{\alpha_i} W_t^{1-\alpha_i}\right)^{\theta_i} \cdot \left(\prod_{j=1}^{N} P_{t,j}^{\omega_{ij}}\right)^{1-\theta_i}. \quad \text{(Sector } i \text{ marginal cost)} \quad (4.9)$$

In the quantitative analysis, we also consider the Leontief production technology, eliminating the substitutability across factors.

4.3 Investment goods

In each sector, investment goods are produced under perfect competition based on a sector-specific CRS technology, which combines sector-specific goods. Investment in sector i is given by

$$I_{t,i} = \bar{\lambda} \prod_{j=1}^{N} I_{t,ij}^{\lambda_{ij}},$$

where $I_{t,ij}$ is sector-j output used to produce investment in sector i, $\bar{\lambda}$ is a normalizing constant. Given the sector-specific investment price $P_{t,i}^I$, a producer of investment goods chooses inputs to maximize profits, yielding the following sector-specific investment demand:

$$P_{t,j}I_{t,ij} = \lambda_{ij}P_{t,i}^I I_{t,i}. \quad \text{(Sector-}i\text{ investment inputs demand)} \quad (4.10)$$

The price of sector-i investment good is then given by

$$P_{t,i}^I = \prod_{j=1}^{N} P_{t,j}^{\lambda_{ij}}. \quad \text{(Sector } i \text{ investment good price)} \quad (4.11)$$

4.4 Sectoral capital accumulation and reallocation

Each sector accumulates sector-specific capital and rents out available capital to output producers in sector i, buy/sell capital from/to other sectors, and produce new capital by buying sector-i investment good. These firms maximize the expected stream of profits:

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} \left[r_{t,i}\hat{K}_{t,i} - P^I_{t,i} I_{t,i} - \sum_{j=1}^{N} P^o_{t,ij} R_{t,ij} \right],$$

where $R_{t,ij}$ capital reallocated from sector j to sector i and $P^o_{t,ij}$ price of this capital; $Q_{0,t}$ is a t-period stochastic discount factor.

While the new investment becomes available with a lag, the reallocated capital becomes available immediately. Let $K_{t-1,i}$ be sector-i capital at the beginning of period t and total capital reallocated towards sector i be given by $R_{t,i} = \sum_{j=1}^{N} R_{t,ij}$, then the capital available for production at time t is

$$\hat{K}_{t,i} = K_{t-1,i} + R_{t,i} - \underbrace{\frac{1}{2} \sum_{j=1}^{N} \phi_{ij} R^2_{t,ij}}_{\text{realloc. cost}} . \quad \text{(Sector } i \text{ available capital)} \tag{4.12}$$

In the expression above, the third term on the RHS captures the reallocation costs paid by sector-i firms for reallocating capital from each sector. Capital accumulation is given by

$$K_{t,i} = (1-\delta)\hat{K}_{t,i} + I_{t,i} \quad \text{(Sector } i \text{ capital accumulation)} \tag{4.13}$$

The first-order conditions of the firms' optimization problem are given by:

$$P^I_{t,i} = E_t Q_{t,t+1} \left[r_{t+1,i} + (1-\delta) P^I_{t+1,i} \right], \qquad \text{(Investment price dyn.)} \tag{4.14}$$

$$P^o_{t,ij} = \left[r_{t,i} + (1-\delta) P^I_{t,i} \right] \cdot (1 - \phi_{ij} R_{ij}). \qquad \text{(Reallocation price)} \tag{4.15}$$

Reallocation between each pair of sectors implies the following reallocation constraints

$$R_{t,ij} = -R_{t,ji} \qquad \text{(Reallocation quantity symmetry)} \tag{4.16}$$

$$P^o_{t,ij} = P^o_{t,ji} \qquad \text{(Reallocation price symmetry)} \tag{4.17}$$

Reallocation demand. The sector-specific price of old capital in sector i is given by

$$P^o_{t,i} = r_{t,i} + (1-\delta) P^I_{t,i}. \quad \text{(Sector-}i \text{ old capital price)} \tag{4.18}$$

Then equation (4.15) becomes $P^o_{t,ij} = P^o_{t,i}(1 - \phi_{ij} R_{ij})$. Then, using the reallocation constraints (4.16) and (4.17), we get the sector-pair specific reallocation as

$$R_{t,ij} = \frac{P^o_{t,i} - P^o_{t,j}}{\phi_{ij} P^o_{t,i} + \phi_{ji} P^o_{t,j}}. \tag{4.19}$$

That is, capital is reallocated from j to i as long as $P^o_{t,i} > P^o_{t,j}$ and the reallocation amount is decreasing in reallocation cost parameters ϕ_{ij} and ϕ_{ji}. Then, the total capital reallocated towards sector i is

$$R_{t,i} = \sum_{j=1}^{N} R_{t,ij} = \sum_{j=1}^{N} \frac{P^o_{t,i} - P^o_{t,j}}{\phi_{ij} P^o_{t,i} + \phi_{ji} P^o_{t,j}}. \quad \text{(Capital reallocation)} \tag{4.20}$$

4.5 Government policy and resource constraint

There is an exogenous stream of government purchases in each sector i, denoted by $G_{t,i}$. The resource constraint on output in sector i implies that

$$Y_{t,i} = C_{t,i} + \sum_{j=1}^{N} X_{t,ji} + \sum_{j=1}^{N} I_{t,ji} + G_{t,i}. \quad \text{(Sector } i \text{ resource constraint)} \tag{4.21}$$

That is, sector-i output is either consumed by household, used as intermediate input in production of output and investment goods, or consumed by the government.

The model is log-linearized around the zero-reallocation steady state, and solved using a standard Blanchard-Khan solution method. The details of the log-linear model and the solution algorithm are available in the Appendix B.

4.6 Analytical characterization of the military multiplier

We now analyze how the equilibrium properties of the military-goods market depend on key features of the model economy. To that end, we derive the demand and supply schedules for military goods under a few simplifying assumptions and show how their elasticities are linked to model parameters. Let y^M_t denote military output, p^M_t the price of military goods, and g^M_t government consumption of military goods; all variables are expressed as percentage deviations from steady state. To obtain demand and supply curves, we impose four assumptions: (i) no input–output links ($L = I$); (ii) labor supply is inelastic and free of wealth effects; (iii) capital depreciates fully each period ($\delta = 1$); and (iv) the capital stock carried over from period $t - 1$ is at its steady-state level. These assumptions yield demand and supply expressions in a simple, intuitive form.

As shown in Section 2, the military multiplier is pinned down by the demand- and supply-side elasticities in this market. The following proposition summarizes how those elasticities vary with the underlying model features.

Proposition 1 (Military good market)**.** *Consider a multi-sectoral efficient economy with Military sector. The elasticity of military demand is:*

$$\epsilon_M^D = \underbrace{\sum_{f \in F} v_f^M \cdot \epsilon_f^D}_{\text{private uses}} \tag{4.22}$$

where F is a set of all non-government (private) uses of military good; v_f^M *is the share of use* f *in total military output;* ϵ_f^D *is demand elasticity of use* f*.*

The elasticity of military good supply is:

$$\epsilon_M^S = \underbrace{\epsilon \cdot \frac{1-\alpha_M}{\alpha_M}}_{\text{factor substitution}} + \underbrace{\frac{1}{\alpha_M \xi_M} \cdot \sum_{j=1}^{N} \frac{\alpha_j \xi_j}{1+\tilde{\phi}_{Mj}} \cdot \epsilon_{Mj}^R}_{\text{capital realloc.}} \tag{4.23}$$

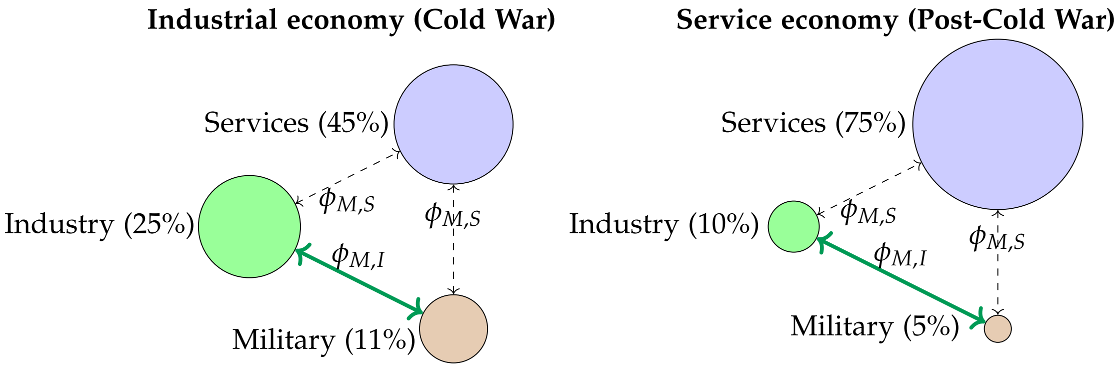
where ϵ *is substitution elasticity between capital and other factors of production;* $\tilde{\phi}_{Mj}$ *is capital reallocation cost from sector* j *to military sector,* ξ_j *size of sector* j *by sales,* α_j *capital share in production of sector* j*,* ϵ_{Mj}^R *is price elasticity of old capital reallocation from sector* j *to military sector.*

See Appendix B.3 for the proof. Demand elasticity rises with the size of the non-government market for military goods, while supply elasticity increases with the degree of factor substitutability. It also grows with the combined size of the sectors from which capital is reallocated, $\alpha_j \xi_j$, and falls with the cost of reallocating capital to the Military sector, $\tilde{\phi}_{Mj}$. In the full-scale model below, we quantify these margins and how they change over time.

5 What determines the military multiplier?

In this section, we show that a three-sector version of the model, when calibrated to the US economy, can account for the observed evidence on the *MM*. By interpreting the evidence on the *MM* through the lens of the model, we are able to study and quantify its determinants. As argued in Section 2, the price elasticity of supply and demand for military goods directly determines the multiplier. However, at a more fundamental level, these elasticities reflect how productive capacity and overall demand adjust across sectors. Therefore, a general equilibrium perspective is warranted. This is what our model provides.

Figure 5: Industry structure in the US



Notes: Size of "Military sector" is computed as share of federal defense spending and defense exports in GDP. The Industry and Services sector sizes are set to the Manufacturing and Services shares in GDP. Computations are based on NIPA tables from BEA. "Cold War" corresponds to the year 1950 (end of year). "Post-Cold War" is the year 2020. Note that sector shares do not sum to 1 (we exclude sectors like Construction and Agriculture). See Appendix B.5 for sources.

5.1 Calibration

We work with a three-sector version of the model and consider the *Military sector* alongside *Industry* and *Services* ($N = 3$). In light of the evidence presented in Section 3 above, we consider two scenarios as we calibrate the model to the US economy: an *industrial economy* and a *service economy*, which are characterized by a large industry and services sector, respectively. Relatively speaking, the US economy during the Cold-War period is still an industrial economy: According to data from the Bureau of Economic Analysis (BEA) Industry accounts for approximately 25 percent and Services for 45 percent of GDP in the 1950s. The post-Cold war economy, instead, is a service economy with Industry accounting for only 10 percent of value added and Services for 75 percent. We treat the Military sector as a separate sector to capture the specific nature of military goods and calibrate its size as the share of federal defense spending and defense exports in GDP, which equals 11% in 1950 and 5% in 2020. Within the Military sector, we equate private consumption of military goods with US arms exports, which have been rising from 0.3% of GDP during the Cold War to 0.8% in 2020.[4]

Figure 5 offers a visual representation, with each circle representing a sector and the connecting arrows indicating the possibility of adjusting capital across

[4]The US state department issues yearly summaries of the dollar amount US arms transfers. The Cold War average is obtained from "Foreign military sales, foreign military construction sales and military assistance facts as of September 30, 1990."

Table 1: Calibration targets and parameter values

Panel A: Impact *MM* to a military buildup

Period	Empirical (%)	Model (%)
Cold War	0.86	0.86
Post Cold War	0.41	0.41

Panel B: Capital reallocation cost parameters

Sector pairs	Reallocation cost parameter ϕ_{ij}
Industry to Military	0.036
Services to Military	26.94

Panel C: Other parameters

Parameter description	Symbol	Value
Depreciation rate	δ	10%
Discount rate	β	0.96
Frisch labor supply elasticity	γ	1
Share of primary factors in production	θ_i	1
Capital share in primary factors	α_i	0.3
Persistence of military spending, AR(2)	ρ_g^1, ρ_g^2	1.4, -0.5
Military good investment technology	$\lambda_{MM}, \lambda_{MI}, \lambda_{MS}$	0, 1, 0

Notes: Panel A: Military multiplier during the first year, see Figure 4. Panel B: Parameter of the quadratic cost function 4.12.

sectors and, hence, the size of sectors. That such adjustments are potentially easier between industry and military is indicated by the solid line. In the model, the costs of adjusting sector sizes depend on how easily capital can be reallocated across sectors, $\phi_{M,I}$ and $\phi_{M,S}$.[5] Ultimately, these costs are reflected in the response of prices to a military expansion, which determine the military multiplier as shown in Section 2. Accordingly, in the calibration, we pin down the reallocation-cost parameters from the *Industry* and *Services* sectors to the *Military* sector by targeting our empirical estimate of impact military multipliers during Cold War and post-Cold War times. The time horizon of the model is one year, implying a time lag of one year for new investments to come online.

We summarize the calibration in Table 1. In the post-Cold War economy, the

[5] The reallocation cost between Services and Industry is equal to the Military-Services cost.

impact military multiplier is only half as large as during the Cold War, see Panel A. Panel B of the same table reports the implied reallocation-cost parameters which are restricted to be constant over time. According to the calibrated model, reallocating capital from the Industry to Military incurs relatively low costs, while shifting capital from the Service results in substantial capital losses—roughly 15% of the reallocated capital is lost. This is intuitive, as productive assets in Industry can be more easily repurposed for military production. For example, converting an automobile manufacturing plant to produce tanks is considerably less costly than converting a restaurant for the same purpose.

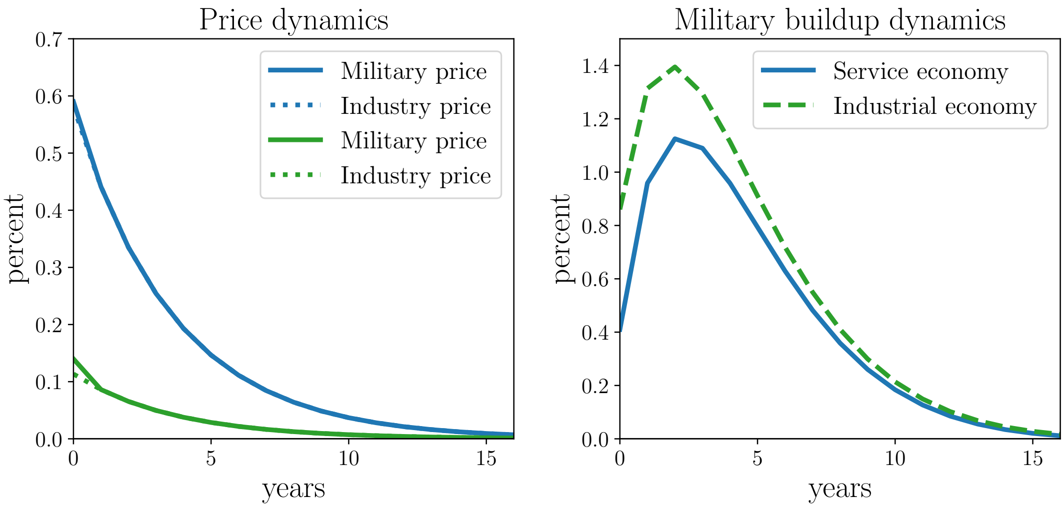
We set the remaining model parameters to the values reported in Panel C of Table 1. Note that the baseline version of the model abstracts from IO links in production (output is produced using primary factors only). Finally, we calibrate the investment network such that the Industry and Services use exclusively their own output to produce sector-specific investment good, in line with evidence by Vom Lehn and Winberry (2022) suggesting that the US investment network is quite concentrated. The military sector, instead, uses output from Industry to produce its investment good.[6] Hence, the size of the Industry sector matters for both capital reallocation and the installation of new capital.

5.2 Military buildup: Model simulation

We simulate a military buildup in the model, assuming an AR(2) process, see again Table 1, for a shock increasing military spending by 1% on impact. In this way we capture the gradual increase in spending over time documented by Ramey (2011). We show the impulse response functions in Figure 6. The horizontal axis measures time in years, the vertical axis measures the deviation from steady state in percent. Prices, shown in the left panel, respond immediately and remain elevated for an extended period for both military and industry goods. The increase in relative prices is much stronger in the Service economy (solid blue line) than in the Industrial economy (dashed green line) because if the industrial base is small, a large part of the increase in military spending is absorbed by an increase in prices, making the buildup less effective. The right panel of Figure 6 shows the increase of actual military goods, illustrating this point: it is considerably larger in the Industrial economy.

[6]This calibration ensures that Military consumption goods and Military investment goods are not perfectly substitutable, and an increase in Military consumption cannot be achieved by decreasing Military investment.

Figure 6: Industry/Military prices and equipment produced

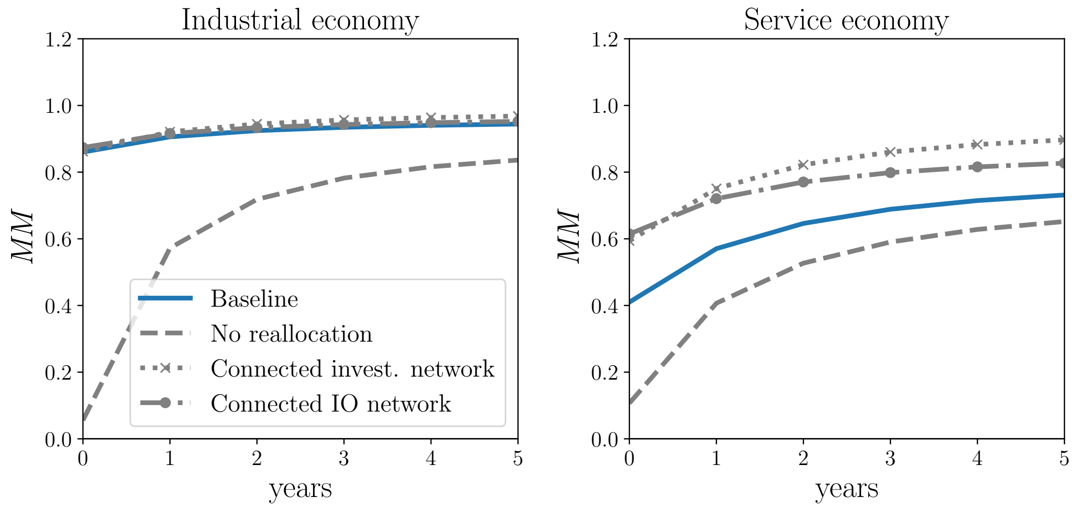


Note: Impulse responses of Military good prices and Industry good prices (left panel) and Equipment produced (right panel) to the military buildup shock in Industrial and Service economies.

Our calibration strategy is tailored for the model to match the evidence on the initial response of the *MM* (first-year response). Still, the model delivers accurate predictions for the dynamics of the military multiplier for both sample periods, based on the two free parameters only. To see this, consider again Figure 4 above. The dashed (blue) lines show the model predictions—they align closely with the evidence. In the Industrial economy (left), the impact multiplier is 0.86, meaning that an increase of government spending by one percentage point of GDP results in an increase in the production of military goods by 0.86 percentage points. In contrast, in the services-based economy, the military multiplier is much lower at 0.41.

Over time, the cumulative *MM* increases in both economies, reflecting the adjustment process that takes place due to new investment and capital reallocation. However, even after several years, the multipliers in both economies do not fully converge. At its peak value, the military multiplier in the Industrial economy is about 1.01, suggesting very effective military spending. In the Service economy, however, the peak multiplier is 0.76 only, meaning that 24% of the overall spending is effectively lost due to the increase of the relative price of military goods.

Figure 7: Military multiplier (cumulative): Counterfactuals



Note: Cumulative *MM* over the first 5 years after the military buildup shock for the Industrial economy (left panel) and Service economy (right panel). **Blue solid** lines plot the baseline model. **Dashed** lines plot the model without capital reallocation. **Dotted** lines plot the model with Military and Industry goods used for investment in all sectors (more connected investment network). **Dotted-dashed** lines plot the model with Military and Industry goods heavily used in the production of Services.

5.3 Economic structure and the military multiplier

Given that the model performs well empirically, we use it to shed light on the factors that shape the *MM*. The main result of our model-based analysis is that as we calibrate the model to the Industrial and the Service economy, respectively, it can fully account for the evidence on the *MM* in the US for two different time periods. The only difference across the two model scenarios is the size of sectors—all other parameters are constant. Hence, the economic structure as reflected in the size of different sectors is key for the *MM*.

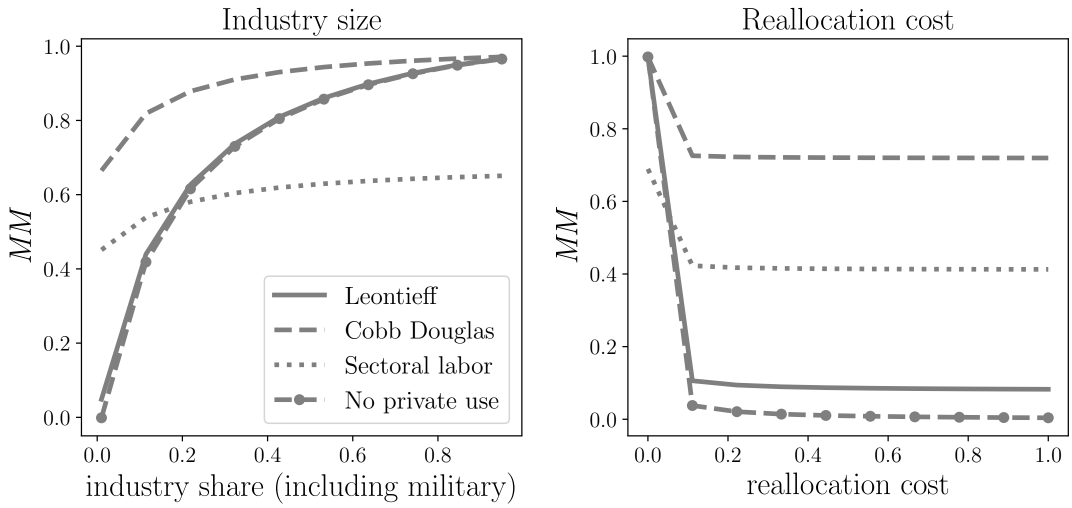
In what follows, we investigate this point systematically through a series of model-based counterfactual experiments. In a first step, we illustrate the role of capital reallocation costs and the role of the production and investment network. Figure 7 shows the results. The solid (blue) lines reproduce the *MM* for the baseline, shown in Figure 4 above. We contrast this baseline with results for an economy where reallocation of capital is prohibitively expensive, indicated by the dashed line. In this case, the impact multiplier is basically zero, since there is no way productive capacity in the military sector can be instantaneously expanded. Over time, however, the military output increases through new investment, and the multiplier becomes larger. The pattern is similar in both panels, although the multiplier remains smaller in the Service economy throughout the time horizons under consideration, just like in the baseline (right panel). We thus see once more that the structure of the economy—given by the size of sectors in steady state—is key for how quickly the productive capacity of the military sector can expand. The Industrial economy is better equipped for this than the Service economy. After all, our calibration implies that repurposing industry-sector capital for military production is cheaper than converting service-sector capital.

To analyze systematically how the industry structure determines the *MM* we consider a range of parameterizations for the size of the industry sector. The left panel of Figure 8 shows how the on-impact military multiplier, measured against the vertical axis, changes with the size of Industry, measured against the horizontal axis. We distinguish four cases. Next to the baseline (solid line), for which we assume a Leontief production function, there is the case of a Cobb-Douglas production function without (dashed line) and with sector-specific labor, meaning labor cannot move across sectors (dotted line). Finally, we consider the case of no private use of Military goods (dashed-dotted line), i.e., no arms exports from the US to the rest of the world.

Across all four settings, the *MM* increases in the steady-state share of Industry. A more industrialized economy can be more easily militarized because reallocating capital from Industry to the Military sector is less costly than from the Services sector. The size of the *MM* is different across the four versions of the model. Our baseline model with Leontieff production function yields the widest range of possible multipliers, depending on industry share because, with flexible labor, capital reallocation is the margin of short-run adjustment that matters. The Cobb-Douglas model generally yields a larger *MM* due the possibility to substitute capital with labor. When we assume sector-specific labor instead, the *MM* declines in the Cobb-Douglas economy.

In the right panel of Figure 8, we zoom in on the role of capital adjustment costs. We still distinguish the four versions of the model and measure the on impact *MM* on the vertical axis and reallocation costs along the horizontal axis. We also see that the *MM* decreases in the capital reallocation costs between the Military and Industry sectors (holding the size of sectors constant). The more difficult it is to reallocate capital, the less effective the military buildup becomes in all versions of the model. Without the private use of Military goods, the *MM*

Figure 8: Military multiplier: The role of industry share and reallocation cost

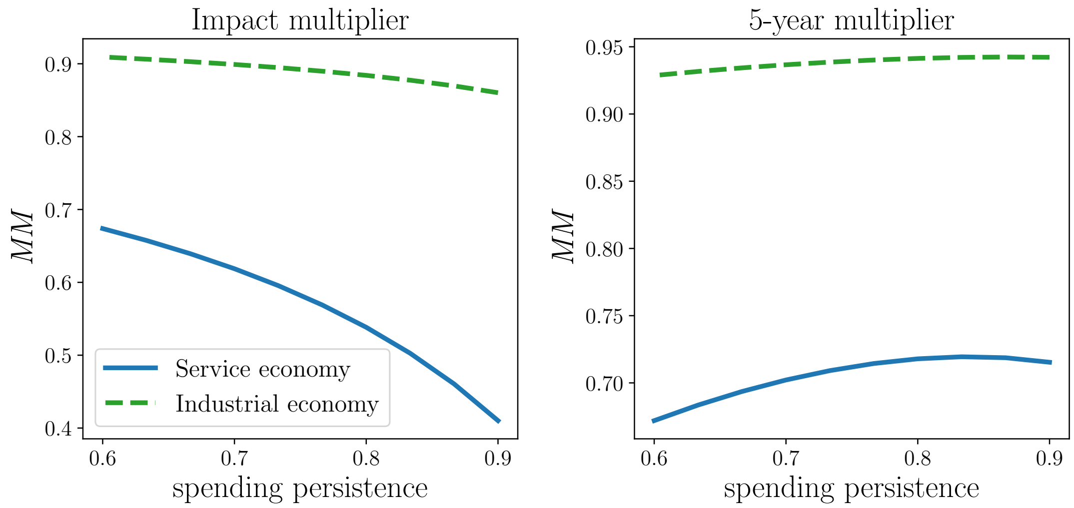


Note: This figure plots how the on-impact *MM* depends on the industry share in GDP (**left panel**), and average reallocation cost in the economy (**right panel**). The **solid line** plots the case of Leontief production technology, the **dashed line** - Cobb-Douglas technology with mobile labor, and the **dotted line** - Cobb-Douglas technology with sector-specific labor, **dashed line with circles** - economy without private consumption of military good.

even approaches zero. In that sense, arms exports provide a lower bound for the multiplier as this production can be redirected for domestic use. If, instead, reallocating capital across sectors can be achieved without costs, the military multiplier is 1, provided labor is mobile across sectors, too.

The *MM* depends also on the network structure. To see this, consider again Figure 7. The dotted and dashed-dotted lines show the *MM* when military and industrial goods are used by the broader economy, either in the form of investment goods or as intermediate inputs. In both cases, the involvement of military/industry in the production of other goods increases the *MM*. This is due to the fact that the effective size of the military-industrial complex as measured by sales (as opposed to final output) becomes larger, which in turn implies that more industrial capital is available for a buildup. This effect is quantitatively small when the *MM* is already large, as in the case of the Industrial economy (left panel), but the effect becomes sizable in the Service economy (right panel). We also investigate the role of the production network more systematically, by varying the use of military goods in industry and vice versa. Overall, we find the quantitative effect on the *MM* more limited, see Figure B.1 in the appendix.

Figure 9: Military multiplier and Buildup persistence



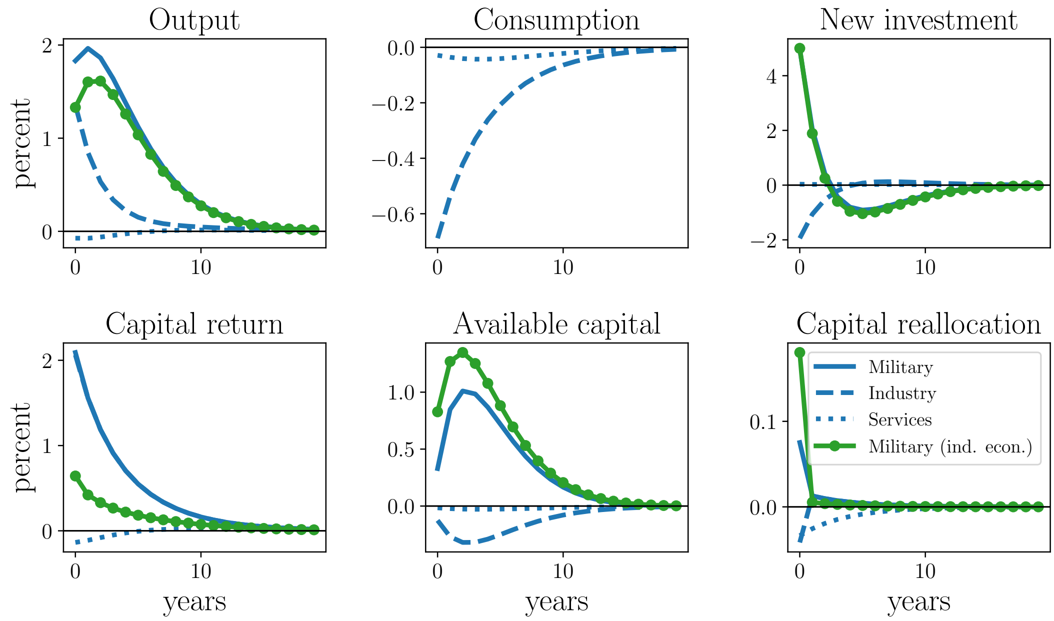
Note: Impact *MM* (left panel) and cumulative *MM* over 5 years (right panel) varying the persistence of military buildups. The solid line plots the Service economy, and the dashed line plots the Industrial economy. Persistence is measured as $\rho_g = \rho_g^1 + \rho_g^2$ (persistence of AR(1) process yielding equivalent long-run cumulative IRF).

This is due to the fact that in a calibrated model, Industry and Military can already relatively easily share factors of production. Hence, increasing the involvement of these two sectors in each other's production does not increase much the overall size of the available military-industrial base. Linkages with the Services sector, however, can lower the *MM*, but this is quantitatively of second-order relative to the role of sector shares and reallocation costs.

5.4 Policy

Finally, we look into how the *MM* depends on the persistence of the buildup. Figure 9 shows the impact multiplier and a cumulative multiplier over 5 years, depending on the persistence of the military spending in both Services and Industrial economies. We see that the impact multiplier is smaller for the more persistent spending, but the cumulative multiplier is larger. This pattern emerges because persistent military spending has a twofold effect: it not only causes an immediate increase in demand for military goods, but also signals that this elevated demand will persist over time. As a result, prices rise more sharply due to higher expected returns on military-related capital. However, over time, as investment increases, the cumulative effect of the spending becomes more efficient and impactful.

Figure 10: Military spending shock: Sectoral response



Notes: Impulse responses of the Military sector (**solid lines**), Industry sector (**dashed lines**), and Services sector (**dotted lines**). The **blue** lines correspond to the response of each sector in the Service economy. The **green o-marked** line shows the response of the Military sector in the Industrial economy. Consumption refers to private consumption by households, and hence excludes consumption of military goods.

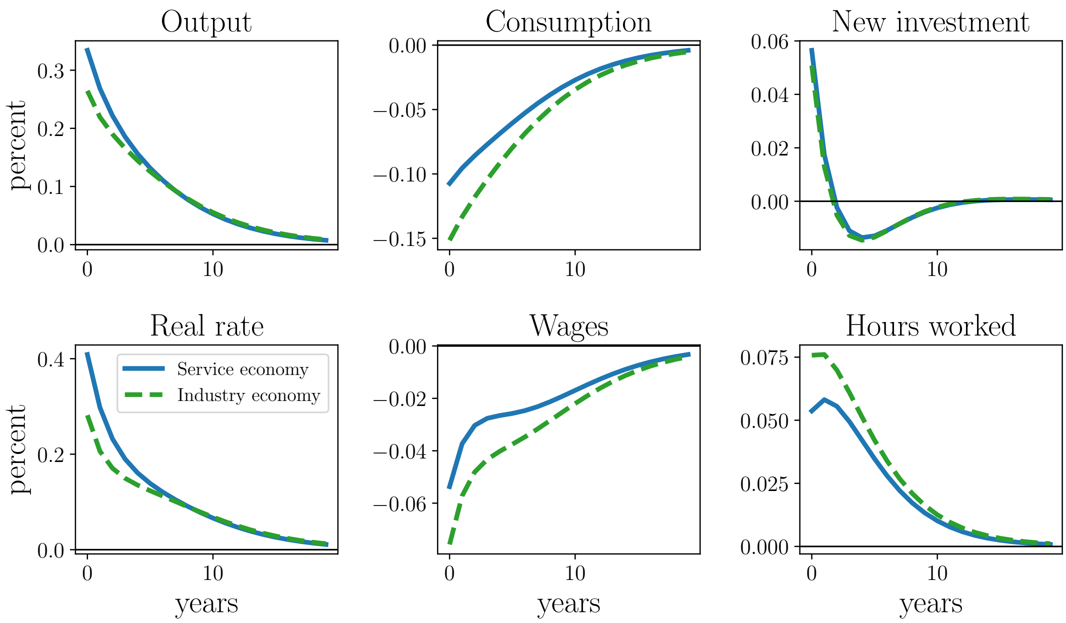
5.5 The transmission mechanism

The effectiveness of military spending ultimately depends on how easily resources can be reallocated across sectors. We now examine in more detail how different sectors respond to a military spending shock. Figure 10 plots the sectoral impulse responses in the Service economy (blue) and compares them with selected responses from the Industrial economy (green).

Following the military buildup, output increases in both, Military and Industry, while it declines in Services. The rise in Military's output is intuitive, as it is directly driven by new government demand. The increase in Industry's output, however, is more nuanced. In our calibration, investment goods for the Military sector also require input from Industry. Therefore, as military investment rises, output in Industry must also increase to supply the necessary goods. At the same time, private consumption of both Industry and Services-goods falls as resources shift away from the private sector toward military production.

Investment and the return on capital in the military sector also rise. While

Figure 11: Aggregate responses: Service v Industrial economy



Notes: Aggregate impulse responses in the Service economy (**blue line**) and Industrial economy (**green line**).

new investment materializes only in the second period, the available capital increases on impact due to reallocation. Importantly, most of this capital is reallocated from Industry rather than from Services, reflecting lower reallocation costs between Industry and Military.

Comparing the sectoral dynamics in the Industrial economy and the Service economy, we find important differences. Although the responses of investment are fairly similar, the patterns of capital reallocation diverge. In the Industrial economy, more capital is reallocated overall, and the return on capital rises less than in the Service economy. This is because the Industrial economy has a larger pool of capital that can be more easily repurposed for military use.

Historically, the macroeconomic literature has focused on the effects of government spending on the broader economy, rather than on specific sectors. While general government spending often aims to stimulate aggregate economic activity, the primary objective of military spending is to generate military output, not necessarily to boost overall GDP. Nevertheless, when pursuing this goal, the broader macroeconomic effects of military spending should remain a consideration for policymakers.

With this in mind, we briefly examine the aggregate response of the model economy to a military buildup shock. Figure 11 shows the response of key aggregate variables in both the Services and Industrial economies. We find that these aggregate responses are qualitatively similar across the two models. In both cases, a military spending shock leads to an increase in aggregate output, indicating that military spending is expansionary at the macroeconomic level. However, private consumption declines, since government consumption must ultimately be financed by private households. At the same time, total hours worked rise. As a result, private households—who are both consumers and workers—are worse off relative to an economy without militarization, to the extent that we abstract from the welfare benefits of higher military capacity.

While new investment initially increases on impact, it subsequently falls below its steady-state level. This occurs because, after the initial boost driven by the Military sector, underinvestment in other sectors begins to dominate at the aggregate level.

Finally, we turn to the central question in the literature on fiscal shocks: the size of the fiscal multiplier. In our model, a 1% increase in military spending raises aggregate output by only 0.06%, implying a fiscal multiplier smaller than 1. This result is not surprising, as small multipliers are a common feature of real business cycle (RBC) models with flexible prices. However, in the context of military spending, the more relevant metric is the military multiplier, not the fiscal multiplier.

6 Conclusion

For the longest time, macroeconomics has been concerned with the business cycle impact of government spending. Changes in military spending, in particular, have been used to study the fiscal multiplier because they arguably vary for reasons exogenous to the business cycle. Ultimately, however, military spending serves a different objective than stabilizing the business cycle. Whether these objectives—external security or geopolitical ends—can be met, depends on economic factors, among other things, how quickly and efficiently economic resources can be mobilized to meet a certain level of military capacity.

In this paper, we put forward the notion of the military multiplier to account for this fact. In the short run, the multiplier can fall significantly below one because allocating resources to military production is costly. We show that these costs depend on initial conditions, such as industrial structure and capital reallocation frictions. We further document, based on military spending shocks, that the military multiplier has declined over time. Using a calibrated multi-sector business cycle model of the US economy, we show that this decline stems from the economy's structural shift toward the service sector.

Future research could build on our framework by quantifying reallocation costs across more disaggregated sectors and constructing capital reallocation networks to better understand sector-specific frictions in mobilizing resources for military production. This would allow for a richer treatment of heterogeneity in adjustment costs and provide more precise estimates of military production efficiency. Cross-country analysis of the military multiplier—accounting for differences in sectoral composition—could shed light on how national industrial structures affect the responsiveness of military capacity to additional spending. Finally, exploring how military procurement could be coordinated within entities like the European Union may reveal how variation in national multipliers shapes the efficiency and strategic logic of joint procurement.

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Appendix

A Empirical Appendix

A.1 Data for local projections

Unless otherwise noted, all series are available at quarterly frequency from 1947Q1 to 2015Q4 from the St. Louis FED - FRED database (mnemonics in parentheses).[7]

Producer price index of manufacturing goods. Producer price index by commodity: durable manufactured goods (discontinued after 2018Q4) (WPUDUR0211) divided by the GDP deflator (GDPDEF). Monthly frequency aggregated to quarterly using the mean.

Producer price index of military manufacturing goods. Manufacturing producer price index by industry: ammunition, except small arms (PCU332993332993) divided by the GDP deflator (GDPDEF). Only available from 1985Q4. Monthly frequency aggregated to quarterly using the mean.

Government spending. Government consumption expenditures and gross investment (GCE) divided by the GDP deflator (GDPDEF).

Output. Nominal GDP (GDP) divided by the GDP deflator (GDPDEF).

Investment. Gross private domestic investment (GPDI) divided by the GDP deflator (GDPDEF).

Consumption. Sum of personal consumption expenditures for nondurable goods (PCND), durable goods (PCDG) and services (PCESV) divided by the GDP deflator (GDPDEF).

Public debt. Market value of gross federal debt (MVGFD027MNFRBDAL) divided by the GDP deflator (GDPDEF).

Inflation. Log-difference of GDP deflator (GDPDEF).

[7] In the quarterly regressions, we use only data until 2015Q4 to have a consistent sample across all dependent variables. The constraining factor is the availability of the military spending shocks of Ramey (2016).

Nominal interest rate. Quarterly average of the effective federal funds rate (FEDFUNDS) until December 2008 and Wu-Xia Shadow federal funds rate afterwards.

Real interest rate. Long-term rate on government bonds (yield on long-term US government securities (LTGOVTBD) until June 2000 and 20-year treasury constant maturity rate (GS20) afterwards) minus log-difference of GDP deflator (GDPDEF) (see Krishnamurthy and Vissing-Jorgensen, 2012).

Military spending shocks. Ramey (2016)-series of narratively-identified defense news shocks. Series available from 1947Q1 to 2015Q4 on Valerie Ramey's homepage (`https://econweb.ucsd.edu/~vramey/research.html`).

A.2 Additional empirical results

Figure A.1: Response of manufacturing and weapon prices to the military buildup shock in the post cold war period

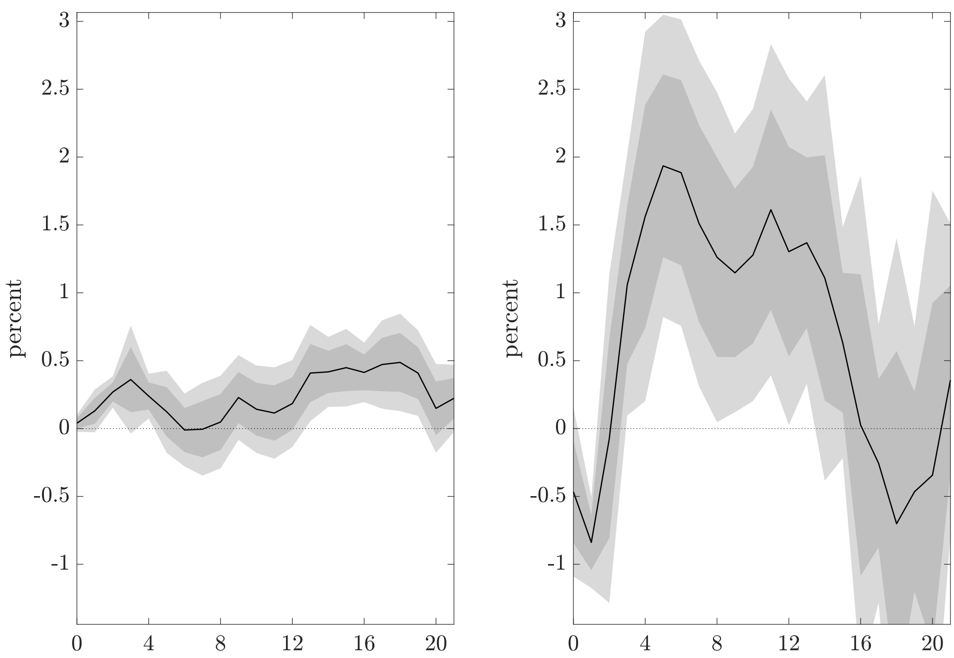
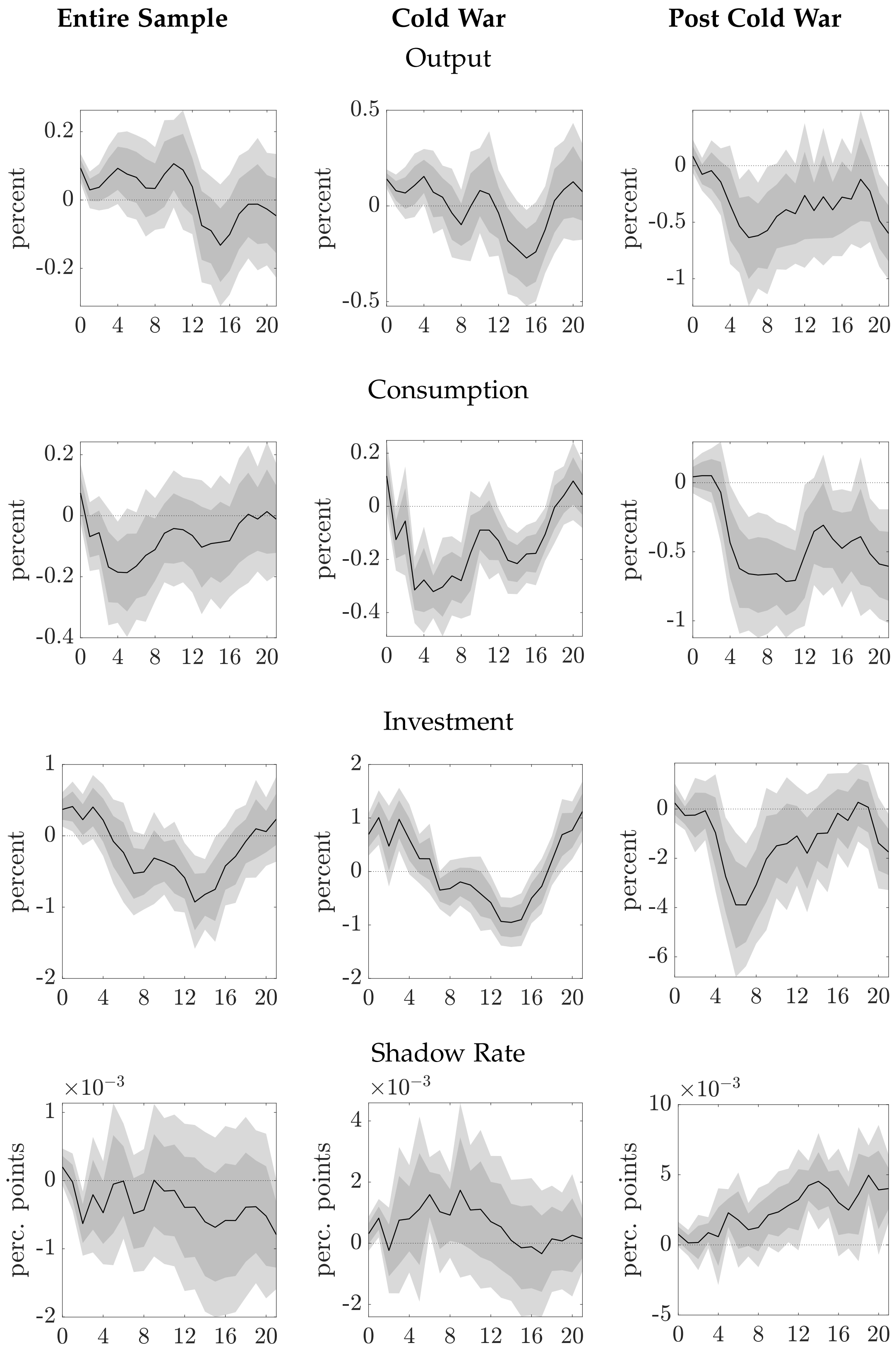
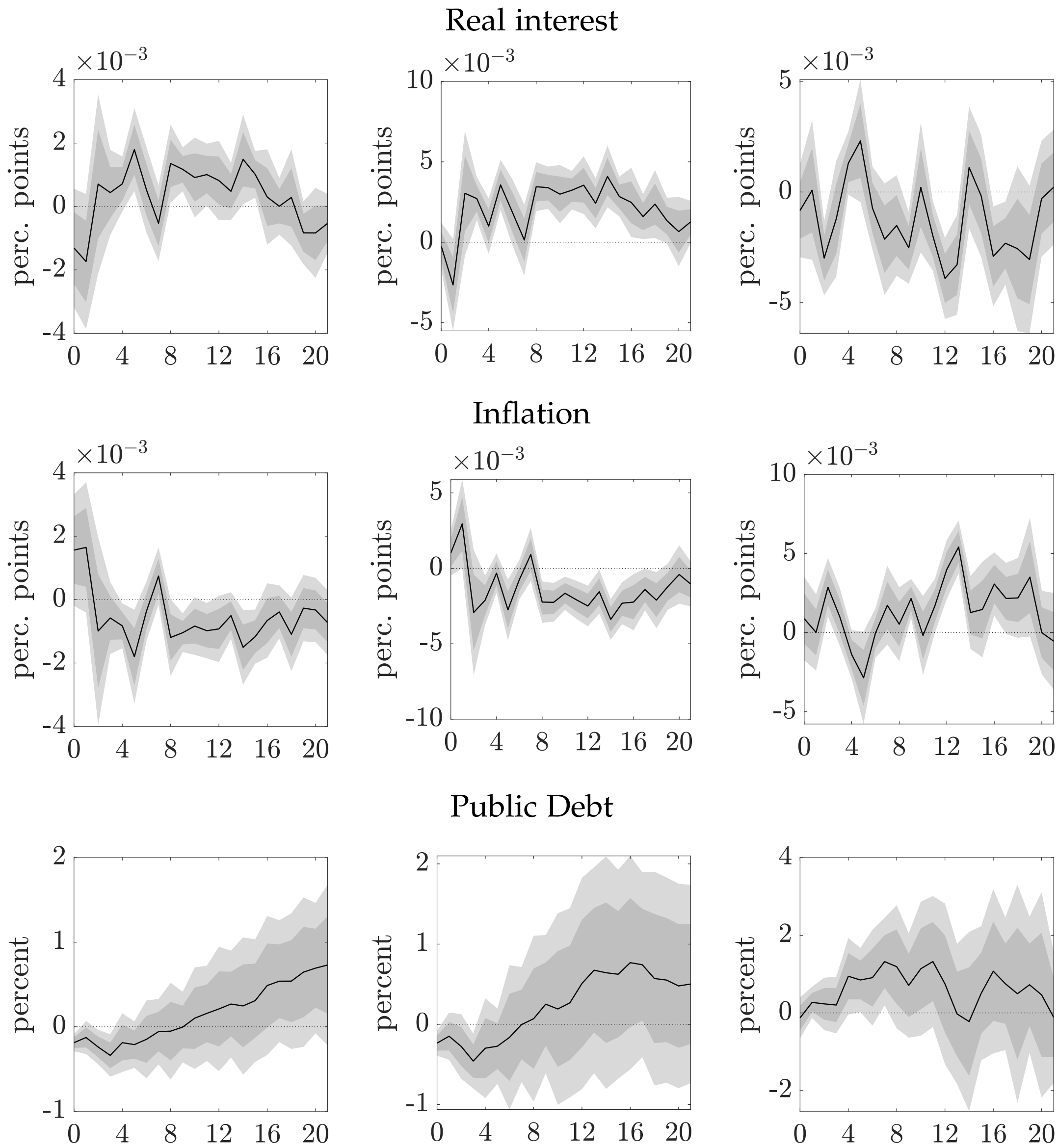


Figure A.2: Empirical responses to fiscal expansion (US) - Additional variables





Notes: Impulse responses to a government spending shock. IRFs based on narrative identification via military news series from Ramey (2016); IRFs for the Post Cold War Period are scaled so that the maximum response of government spending equals that of the Cold War period. Light (dark) gray areas are 90 percent (68 percent) confidence bounds based on **?**-standard errors.

B Model Appendix

B.1 Log-linearization

The model solution relies on log-linearization. We log-linearize the model around a steady state in which $\bar{P}_i = 1$ for all i (this normalization is arbitrary). Let us first introduce some notations.

Notation. Bar letter $\bar{X}$ denote steady state value of X_t. Small letter denotes log-deviation from steady state $x_t = log(X_t) - log(\bar{X})$. Bold letters denote to column vectors of sector-specific variables or parameters $\boldsymbol{x_t} = [x_{t,1}, \ldots x_{t,n}]'$; $\mathbf{1}$ denotes a column vector of ones. Matrix I_x denotes a diagonal matrix with vector $\boldsymbol{x}$ on the main diagonal; I denotes an identity matrix. Let the input-output matrix be denoted as W, such that $W(i,j) = \omega_{ij}$. The investment production matrix is denoted as W_λ, such that $W_\lambda(i,j) = \lambda_{ij}$.

Steady state. Since $\bar{P}_i = 1$ for all i, from 4.11 we have that $\bar{P}_i^I = 1$ for all i. From 4.1 we have that $\bar{Q} = \beta$. Then from 4.14 we have $\bar{r}_i = \frac{1}{\beta} - (1-\delta) = \bar{r}$ for all i. This implies that $\bar{P}_i^o = \bar{P}_j^o$ and that $\bar{R}_{ij} = 0$ for all i and j. Hence, there is *no reallocation in steady state*. Given the absence of reallocation, we have $\bar{I}_i = \delta \bar{K}_i = \delta \cdot \frac{\alpha_i \theta_i}{\bar{r}} \cdot \bar{P}_i \bar{Y}_i$ where the last equality follows from 4.6.

Now let us define the steady state sales shares (Domar weights) as $\xi_i = \frac{\bar{P}_i \bar{Y}_i}{\bar{P}\bar{C}} = \frac{\bar{Y}_i}{\bar{C}}$. Let us also define the steady state government spending shares as $\bar{g}_i = \frac{\bar{G}_i}{\bar{C}}$. Then, taking the resource constraint 4.21, multiplying by $\bar{P}_i$ and using equations 4.8 and 4.10, we obtain $\bar{P}_i \bar{Y}_i = \bar{P}_i \bar{C}_i + \sum_{j=1}^{N} (1-\theta_j) \omega_{ji} \bar{P}_j \bar{Y}_j + \sum_{j=1}^{N} \lambda_{ji} \frac{\delta}{\bar{r}} \alpha_j \theta_j \bar{P}_j \bar{Y}_j + \bar{P}_i \bar{G}_i$, which by dividing by $\bar{C}$ and rearranging yields the vector of sales shares $\boldsymbol{\xi} = [I - W'(I - I_\theta) - \frac{\delta}{\bar{r}} W_\lambda' I_\alpha I_\theta]^{-1} \cdot (\boldsymbol{b} + \bar{\boldsymbol{g}})$ where $\boldsymbol{b}$ is vector of consumption share parameters, $\bar{\boldsymbol{g}}$ is vector of steady state government spending shares. Then, the steady state labor shares are $\frac{\bar{W}\bar{L}_i}{\bar{C}} = (1-\alpha_i)\theta_i \cdot \xi_i$ and investment shares are $\frac{\bar{I}_i}{\bar{C}} = \delta \cdot \frac{\alpha_i \theta_i}{\bar{r}} \cdot \xi_i$.

Next we proceed with log-linearization. Log-linear Euler equation and labor supply are

$$q_{t,t+1} = E_t[c_t - c_{t+1}] \qquad \text{(Euler equation)} \qquad \text{(B.1)}$$

$$\gamma l_t = w_t - c_t \qquad \text{(Labor supply)} \qquad \text{(B.2)}$$

Sectoral consumption demand and consumer price index are

$$\boldsymbol{p_t} + \boldsymbol{c_t} = c \cdot \mathbf{1} \qquad \text{(Sectoral consumption demand)} \tag{B.3}$$

$$\boldsymbol{b'}\boldsymbol{p_t} = 0 \qquad \text{(Consumer price index)} \tag{B.4}$$

Sectoral labor supply aggregation is

$$[\boldsymbol{\xi'}(I - I_\alpha)I_\theta \mathbf{1}] \cdot l_t = \boldsymbol{\xi'}(I - I_\alpha)I_\theta \cdot \boldsymbol{l_t} \quad \text{(Sectoral labor aggregation)} \tag{B.5}$$

Labor demand and capital demand are

$$\boldsymbol{r_t} + \boldsymbol{\hat{k}_t} = \boldsymbol{p_t} + \boldsymbol{y_t} \qquad \text{(Capital demand)} \tag{B.6}$$

$$w_t \mathbf{1} + \boldsymbol{l_t} = \boldsymbol{p_t} + \boldsymbol{y_t} \qquad \text{(Labor demand)} \tag{B.7}$$

Sectoral prices (marginal cost) are

$$\boldsymbol{p_t} = -L\boldsymbol{a_t} + LI_\theta I_\alpha \boldsymbol{r_t} + \cdot L(I - I_\alpha)I_\theta \cdot \mathbf{1} \cdot w_t \quad \text{(Output prices)} \tag{B.8}$$

where $L = [I - W(I - I_\theta)]^{-1}$. Investment prices are

$$\boldsymbol{p_t^I} = W_\lambda \boldsymbol{p_t} \quad \text{(Investment prices)} \tag{B.9}$$

Capital reallocation and accumulation is

$$\boldsymbol{\hat{k}_t} = \boldsymbol{k_{t-1}} + \bar{r}[I_\alpha I_\theta I_\xi]^{-1}\boldsymbol{k_t^r} \qquad \text{(Available capital)} \tag{B.10}$$

$$\boldsymbol{k_t} = (1 - \delta)\boldsymbol{\hat{k}_t} + \delta \boldsymbol{i_t} \qquad \text{(Capital accumulation)} \tag{B.11}$$

where $k_{t,i}^r = \frac{R_{t,i}}{\bar{C}}$ is the reallocated capital as the share of steady state consumption. Investment price dynamics is

$$\boldsymbol{p_t^I} = E_t[q_{t,t+1} \cdot \mathbf{1} + (1 - \beta(1 - \delta)) \cdot \boldsymbol{r_{t+1}} + \beta(1 - \delta)\boldsymbol{p_{t+1}^I}] \tag{B.12}$$

Price of existing capital is

$$\boldsymbol{p_t^o} = \bar{r} \cdot \boldsymbol{r_t} + (1 - \delta)\boldsymbol{p_t^I} \tag{B.13}$$

Log-linearizing sector-pair reallocation, we get $k_{t,ij}^r = \frac{\bar{C}^{-1}}{\phi_{ij}+\phi_{ji}} \cdot (p_{t,i}^o - p_{t,j}^o)$. Then, sectoral reallocation is $k_{t,i}^r = \sum_j k_{t,ij}^r = p_{t,i}^o \sum_j \tilde{\phi}_{ij} - \sum_j \tilde{\phi}_{ij} p_{t,j}^o$ where $\tilde{\phi}_{ij} = \frac{\bar{C}^{-1}}{\phi_{ij}+\phi_{ji}}$. Then the link between sectoral prices of existing capital and sectoral reallocation are

$$\boldsymbol{k_t^r} = (I_\phi - W_\phi) \cdot \boldsymbol{p_t^o} \quad \text{(Capital reallocation)} \tag{B.14}$$

where matrix W_ϕ is such that $W_\phi i, j = \tilde{\phi}_{ij} = \frac{\bar{C}^{-1}}{\phi_{ij}+\phi_{ji}}$ and $I_\phi = diag\{W_\phi \cdot \mathbf{1}\}$. Finally, the resource constraint is

$$[I - W'(I - I_\theta)] \cdot I_\xi \cdot (\boldsymbol{p_t} + \boldsymbol{y_t}) = I_b \mathbf{1} \cdot c_t + \frac{\delta}{\bar{r}} W_\lambda' I_\alpha I_\theta I_\xi \cdot (\boldsymbol{p_t^I} + \boldsymbol{i_t}) + I_g \cdot (\boldsymbol{p_t} + \boldsymbol{g_t}) \tag{B.15}$$

B.2 Model solution

B.2.1 Reduced model system

To solve the model using BK method, we first simplify it to reduce the number of variables. This yields a system consisting of equations[8]

$$W_\lambda \boldsymbol{p_t} = E_t[(c_t - c_{t+1}) \cdot \mathbf{1} + (1-\beta(1-\delta)) \cdot \boldsymbol{r_{t+1}} + \beta(1-\delta) W_\lambda \boldsymbol{p_{t+1}}]$$
$$\boldsymbol{k_t} = (1-\delta)\boldsymbol{\hat{k}_t} + \delta \boldsymbol{i_t}$$
$$[I - W'(I - I_\theta)] \cdot I_\xi \cdot (\boldsymbol{r_t} + \boldsymbol{\hat{k}_t}) = I_b \mathbf{1} \cdot c_t + [\frac{\delta}{\bar{r}} W_\lambda' I_\alpha I_\theta I_\xi W_\lambda + I_g] \cdot \boldsymbol{p_t} + \frac{\delta}{\bar{r}} W_\lambda' I_\alpha I_\theta I_\xi \cdot \boldsymbol{i_t} + I_g \cdot \boldsymbol{g_t}$$
$$\boldsymbol{p_t} = -L\boldsymbol{a_t} + L I_\theta I_\alpha \boldsymbol{r_t} + L(I - I_\alpha) I_\theta \cdot \mathbf{1} \cdot w_t$$
$$\boldsymbol{\hat{k}_t} = \boldsymbol{k_{t-1}} + \bar{r}[I_\alpha I_\theta I_\xi]^{-1} \boldsymbol{k_t^r}$$
$$\boldsymbol{k_t^r} = (I_\phi - W_\phi) \cdot [\bar{r} \cdot \boldsymbol{r_t} + (1-\delta) W_\lambda \boldsymbol{p_t}]$$
$$w_t = \frac{1}{1+\gamma} \cdot c_t + \frac{\gamma}{1+\gamma} \cdot \mathbf{1'}(I - I_\alpha) I_\theta \cdot (\boldsymbol{r_t} + \boldsymbol{\hat{k}_t})$$
$$\boldsymbol{b'} \boldsymbol{p_t} = 0$$

The first two systems are dynamic equations (contain next period variables). The rest are static equations. This system is complemented by the dynamics of the exogenous variables $\boldsymbol{a_t}$ and $\boldsymbol{g_t}$[9]

$$\boldsymbol{a_t} = \rho_a \boldsymbol{a_{t-1}} + \boldsymbol{\epsilon_t^a}$$
$$\boldsymbol{g_t} = \rho_g \boldsymbol{g_{t-1}} + \boldsymbol{\epsilon_t^g}$$

where $\boldsymbol{\epsilon_t^a}$ and $\boldsymbol{\epsilon_t^g}$ are exogenous shocks. The variables in the reduced system are: $\boldsymbol{a_t}$, $\boldsymbol{g_t}$, $\boldsymbol{k_{t-1}}$, $\boldsymbol{r_t}$, $\boldsymbol{\hat{k}_t}$, $\boldsymbol{i_t}$, $\boldsymbol{k_t^r}$, $\boldsymbol{p_t}$, c_t, w_t.

B.2.2 Model solution algorithm

Let $\boldsymbol{x_t}$ be a vector of variables. The system can be written as $A^0 E_t \boldsymbol{x_{t+1}} = A^1 \boldsymbol{x_t}$. Variables in $\boldsymbol{x_t}$ can be partitioned into dynamic variables $\boldsymbol{x_t^d}$ and static variables $\boldsymbol{x_t^s}$, that is $\boldsymbol{x_t} = [\boldsymbol{x_t^d};\ \boldsymbol{x_t^s}]$. For static variables we have $A_{21}^0 = 0$ and $A_{22}^0 = 0$. Then, we have two underlying systems

$$A_{11}^0 E_t \boldsymbol{x_{t+1}^d} + A_{12}^0 E_t \boldsymbol{x_{t+1}^s} = A_{11}^1 \boldsymbol{x_t^d} + A_{12}^1 \boldsymbol{x_t^s}$$
$$0 = A_{21}^1 \boldsymbol{x_t^d} + A_{22}^1 \boldsymbol{x_t^s}$$

[8] The investment matrix W_λ should be full rank to ensure one-to-one mapping.

[9] Alternative (more realistic) government spending process is $\boldsymbol{g_t} = \rho_g^1 \boldsymbol{g_{t-1}} + \rho_g^2 \boldsymbol{g_{t-2}} + \boldsymbol{\epsilon_t^g}$ gives the hump-shaped government spending; requires extending state variables with $\boldsymbol{g_t^l} = \boldsymbol{g_{t-1}}$

Then, static variables can be mapped from dynamic variables as $\boldsymbol{x_t^s} = -[A_{22}^1]^{-1} A_{21}^1 \boldsymbol{x_t^d}$. Substituting for static variables yields the following system

$$(A_{11}^0 - A_{12}^0 \cdot [A_{22}^1]^{-1} A_{21}^1) E_t \boldsymbol{x_{t+1}^d} = (A_{11}^1 - A_{12}^1 \cdot [A_{22}^1]^{-1} A_{21}^1) \boldsymbol{x_t^d}$$

which yields the standard system for BK method

$$E_t \boldsymbol{x_{t+1}^d} = A \boldsymbol{x_t^d}$$

where $A = (A_{11}^0 - A_{12}^0 \cdot [A_{22}^1]^{-1} A_{21}^1)^{-1} \cdot (A_{11}^1 - A_{12}^1 \cdot [A_{22}^1]^{-1} A_{21}^1)$. The dynamic variables are then partitioned into the state variables $\boldsymbol{x_t^{d,s}}$ and jump variables $\boldsymbol{x_t^{d,j}}$, that is $\boldsymbol{x_t^d} = [\boldsymbol{x_t^{d,s}};\ \boldsymbol{x_t^{d,j}}]$. Then solution follows standard BK method, resulting in the solution

$$\begin{aligned} \boldsymbol{x_{t+1}^{d,s}} &= M \boldsymbol{x_t^{d,s}} + \boldsymbol{u_{t+1}} \\ \boldsymbol{x_t^{d,j}} &= G \boldsymbol{x_t^{d,s}} \end{aligned}$$

where $\boldsymbol{u_{t+1}}$ is a vector of exogenous shocks.

In our system $\boldsymbol{x_t^{d,s}} = [\boldsymbol{a_t};\ \boldsymbol{g_t};\ \boldsymbol{k_{t-1}}]$ and $\boldsymbol{x_t^{d,j}} = \boldsymbol{r_t}$. Other variables are static.

B.3 Theoretical appendix

Proposition 2 (Military good market)**.** *Consider a multi-sectoral efficient economy with Military sector. The elasticity of military demand is:*

$$\epsilon_M^D = \underbrace{\sum_{f \in F} v_f^M \cdot \epsilon_f^D}_{\text{private uses}}$$

where F is a set of all non-government (private) uses of military good; v_f^M is the share of use f in total military output; ϵ_f^D is demand elasticity of use f.

The elasticity of military good supply is:

$$\epsilon_M^S = \underbrace{\epsilon \cdot \frac{1-\alpha_M}{\alpha_M}}_{\text{factor substitution}} + \underbrace{\frac{1}{\alpha_M \xi_M} \cdot \sum_{j=1}^{N} \frac{\alpha_j \xi_j}{1 + \tilde{\phi}_{Mj}} \cdot \epsilon_{Mj}^R}_{\text{capital realloc.}}$$

where ϵ is substitution elasticity between capital and other factors of production; $\tilde{\phi}_{Mj}$ is capital reallocation cost from sector j to military sector, ξ_j size of sector j by sales, α_j capital share in production of sector j, ϵ_{Mj}^R is price elasticity of old capital reallocation from sector j to military sector.

Proof. We prove that the stated expressions hold by derivation. To derive demand link we start with the sectoral resource constraint for military sector:

$$Y_{t,i} = C_{t,i} + \sum_{j=1}^{N} X_{t,ji} + \sum_{j=1}^{N} I_{t,ji} + G_{t,i}$$

with $i = M$. Denoting steady state use shares as v_f^M and log-linearizing around the steady state, we get:

$$y_t^M = v_C^M \cdot c_t^M + \sum_{j=1}^{N} v_{Mj}^X \cdot x_t^{jM} + \sum_{j=1}^{N} v_{Mj}^I \cdot i_t^{jM} + v_G^M \cdot g_t^M$$

Let the private demand for each use be be given by $x_t^{jM} = -\epsilon_{jM,X}^D \cdot p_t^M$ (same for consumption and investment uses). Then, substituting these private demand functions into the resource constraints and rearranging, yields the demand demand for military good is:

$$y_t^M = -[\sum_{f \in F} v_f^M \cdot \epsilon_f^D] \cdot p_t^M + v_G^M \cdot g_t^M$$

where $F = \{C, ..X_j.., ..I_j..\}$ is a set of all private uses of military good: private consumption, intermediate goods, and investment goods. v_f^M is the share of use f of military good in total military output (ex. $v_C^M = \frac{C_M}{Y_M}$ is ratio of private consumption of military goods to its output); ϵ_f^D is demand elasticity of f-th use of military good. The elasticity of military demand is:

$$\epsilon_M^D = \sum_{f \in F} v_f^M \cdot \epsilon_f^D$$

Hence, elasticity ϵ_M^D increases with the size of the private market as given by private use shares v_f^M and demand elasticity of each private use ϵ_f^D.

To derive supply equation, let us assume the arbitrary elasticity of substitution between factors of production ϵ. Then, demand for military capital can be written as

$$\hat{k}_t^M = -\epsilon \cdot (r_t^M - p_t^M) + y_t^M$$

Under simplifying assumptions of no input-output network $L = I$ and inelastic labor supply without wealth effect $w_t = 0$, and using the marginal cost expression, we can write

$$p_t^M = \alpha_M \cdot r_t^M$$

Substituting for interest rate, we obtain link between capital and price:

$$\hat{k}_t^M = -\epsilon \cdot (\frac{1}{\alpha_M} - 1) \cdot p_t^M + y_t^M$$

Under the simplifying assumption of full depreciation $k^M_{t-1} = 0$, we have the link between capital reallocation and available capital:

$$\hat{K}^M_t - K^M_0 = \sum R_{Mj,t} - \frac{\phi_{jM}}{2} R^2_{Mj,t} = \sum (1 - \frac{\phi_{jM}}{2} R_{Mj,t}) \cdot R_{Mj,t} \approx \sum \frac{R_{Mj,t}}{1 + \tilde{\phi}_{Mj}}$$

where last approximate relationship stems from the fact that for small x we have $1 - x \approx \frac{1}{1+x}$ and setting $\tilde{\phi}_{jM} = \frac{\phi_{jM} R_{Mj,t}}{2}$ the reallocation cost of given reallocation size. Reallocation in other sectors can be written as: $R_{Mj,t} \approx \hat{K}^j_t - K_0$, that is reallocated capital from sector j to Military should be equal to the change of capital in j.Then, using log-linearization we get: $k^r_{Mj,t} = \alpha_j \xi_i \hat{k}^j_t$ where $\alpha_j \xi_i = \frac{K_i}{C}$ is capital share in consumption. Using this expression, we obtain:

$$\hat{k}^M_t \approx \frac{1}{\alpha_M \xi_M} \sum \frac{\alpha_j \xi_j}{1 + \tilde{\phi}_{Mj}} \hat{k}_{t,j}$$

Finally, let the supply of capital for sector j be given be $\hat{k}_{j,t} = \epsilon^R_{Mj} \cdot p^M_t$. From these relationships we get the military good supply as:

$$y^M_t = \left[\underbrace{\epsilon \cdot \frac{1 - \alpha_M}{\alpha_M}}_{\text{factor substitution}} + \underbrace{\frac{1}{\alpha_M \xi_M} \sum_{j=1}^{N} \frac{\alpha_j \xi_j}{1 + \tilde{\phi}_{Mj}} \cdot \epsilon^R_{Mj}}_{\text{capital realloc.}} \right] \cdot p^M_t$$

where ϵ is substitution elasticity between capital and other factors of production, α_M is capital share of in military output; $\tilde{\phi}_{Mj}$ is reallocation cost from sector j to military sector $\xi_j = \frac{Y_j}{C}$ sales of sector j normalized by total private consumption, α_j capital share in production of sector j, $\bar{r}$ is steady state interest rate, ϵ^R_{Mj} is price elasticity of old capital supply from sector j to military sector.

Elasticity of military good supply is:

$$\epsilon^S_M = \underbrace{\epsilon \cdot \frac{1 - \alpha_M}{\alpha_M}}_{\text{factor substitution}} + \underbrace{\frac{1}{\alpha_M \xi_M} \cdot \sum_{j=1}^{N} \frac{\alpha_j \xi_j}{1 + \tilde{\phi}_{Mj}} \cdot \epsilon^R_{Mj}}_{\text{capital realloc.}}$$

Hence, this elasticity increases in the substitutability of capital with other factors of production ϵ. Second, it decreases with reallocation costs from each sector $\tilde{\phi}_{Mj}$ and increasing with the amount of capital in each of these sectors $\alpha_j \xi_j = \frac{K_j}{C}$ (normalized by total consumption). Also it increases with the price elasticity of supply of used capital from each sector to military sector.

□

B.4 Derivation of cumulative military multiplier

Cumulative multiplier derivation: Let $\{X_t\}_t^h$ be path of gov. spending after the shock. Then, total spending over h periods is $\sum_{t=1}^{h} X_t \approx \bar{X} \sum_{t=1}^{h} x_t$, where x_t is a percentage deviation from pre-shock value $\bar{X}$. Similarly, military equipment produced during these h periods is $\sum_{t=1}^{h} G_t \approx \bar{G} \sum_{t=1}^{h} g_t$. Dividing cumulative spending by cumulative military output yields the cumulative multiplier: $MM = \frac{\sum_{t=1}^{h} G_t}{\sum_{t=1}^{h} X_t} \approx \frac{\sum_{t=1}^{h} g_t}{\sum_{t=1}^{h} x_t}$. We further have, using $X_t = P_t \cdot G_t$,

$$\sum_{j=0}^{h} x_{t+j} = \sum_{j=0}^{h} g_{t+j} + \sum_{j=0}^{h} p_{t+j}.$$

Thus, the cumulative military multiplier can be written as

$$MM(h) = 1 - \frac{\sum_{j=0}^{h} p_{t+j}}{\sum_{j=0}^{h} x_{t+j}},$$

which we use to compute the cumulative multiplier directly from the data.

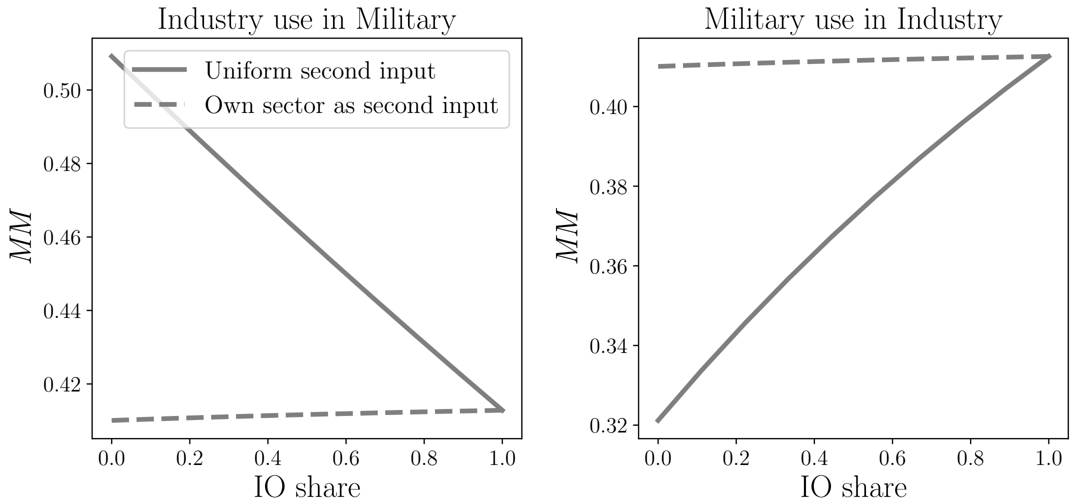
B.5 Calibration appendix

Industry and Service shares. We calibrate Manufacturing and Services shares using "Value Added by Industry as a Percentage of Gross Domestic Product" table from U.S. Bureau of Economic Analysis. For the "Cold-War" economy we take year 1950 and use historical tables. For the "post-Cold-War" economy we take year 2020. We calibrate Industry sector size as a share of Manufacturing in GDP. We compute Services sector size as a sum of shares of the following sectors: utilities, wholesale trade, retail trade, transportation and warehousing, information, finance, insurance, real estate, rental, and leasing, professional and business services, educational services, health care, and social assistance, arts, entertainment, recreation, accommodation, and food services, and other services, except government.

Military sector shares used by the government. Size of military sector is calibrated using the "Table 1.1.5. Gross Domestic Product" from U.S. Bureau of Economic Analysis. We compute Military sector size as a ratio of Federal National defense Spending and the GDP. For the "Cold War" economy, we take the average between 1950 and 1960 to account for the sharp increase in spending when the Korean War started (during this period, the military spending changed from 7.5% to 12.5%). For the "post-Cold War" economy, we take the value in 2020.

B.6 Additional model results

Figure B.1: M-multiplier: Role of input-output network



Notes: This figure shows the sensitivity of on-impact M-multiplier to the input-output structure. **Left panel:** plots how M-multiplier depends on the use of Industry goods in the Military sector. **Right panel:** shows the dependence of the M-multiplier on the use of Military in Industry.

Figure B.1 reports two experiments. **(1) Mixed-input case.** The "second input" is a uniform bundle that always includes Services (Military + Services in the left panel; Industry + Services in the right). **(2) Own-input case.** The second input is the sector's own good (Military on the left, Industry on the right).

Mixed-input. Because Services are always required, linkages arise between the military–industrial complex and the rest of the economy. A larger *industry share in military production* lowers the multiplier: reallocating capital out of industry is harder when industry output is itself needed for military goods. By contrast, a larger *military share in industry production* raises the multiplier, since private (industrial) demand for military inputs is crowded out when government military demand rises.

Own-input. With no cross-sector use, spillovers vanish. Increasing the military input share automatically reduces the industry-input share one-for-one, leaving total military–industrial usage unchanged; the multiplier is therefore flat.